Mercury's Rotation Equations

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November 15, 2004 - Updated July 2012
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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Mercury	Sun
Mass (kg)	$m := 3.3022 \cdot 10^{23}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho := 5.427$	$\rho_s := 1.408$
Axis Tilt (deg)	$t := 0.01$	$t_s := 7.25$
Semi-major Axis (km)	$a := 57910000$	
Orbit Eccentricity (deg)	$e := 0.20563$	
Orbit Inclination (degree), with respect to equator	$i := 3.38$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$\omega_F = 0.78228985$ Mercury's Free Rotation (per day)

Part 1

Mercury's Influenced Rotation by the influence of the Sun

$q := a\cdot(1-e)$

$q = 46001966.7$ Mercury's Perihelion Distance (km)

$Q := a\cdot(1+e)$

$Q = 69818033.3$ Mercury's Aphelion Distance (km)

$$i_r := \left(\left|cos\left(\frac{i\cdot\pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 0.99913023$ Mercury's Influenced Rotation Reduction Factor by Orbit Inclination

$$\omega_{Mi} := \frac{\sqrt[6]{m\cdot i_r \div M \div \sqrt[6]{\rho}}}{i_M}$$

$\omega_{Mi} = 0.44015742$ Mercury's Maximum Influenced Rotation by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m\cdot i_r \div M}}{i_{St}}$$

$S_t = 73233584091.2$ Mercury's Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m\cdot i_r \div M}}{i_{Ma}}$$

$M_a = 130123256.1$ Mercury's Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m\cdot i_r \div M}}{i_{Sp}}$$

$S_p = 69360434.2$ Mercury's Stop Rotation Distance to the Sun (km)

Calculating Mercury's average distance to the Sun, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a}{e}, 0\right), 0\right)$$

$x = 55684648.08$ X value at Mercury's orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 56672452.2$ Mercury's Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 15559538.99$ Y value at the Mercury's orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 0.27247303$ Half-angle of the Mercury's orbit out of S_p Boundary (rad)

$$P_o := 2 \cdot a \cdot \int_0^{\pi} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

$P_o = 359981888$ Mercury's Orbital Perimeter (km)

$$s := a \cdot \int_0^{\theta} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

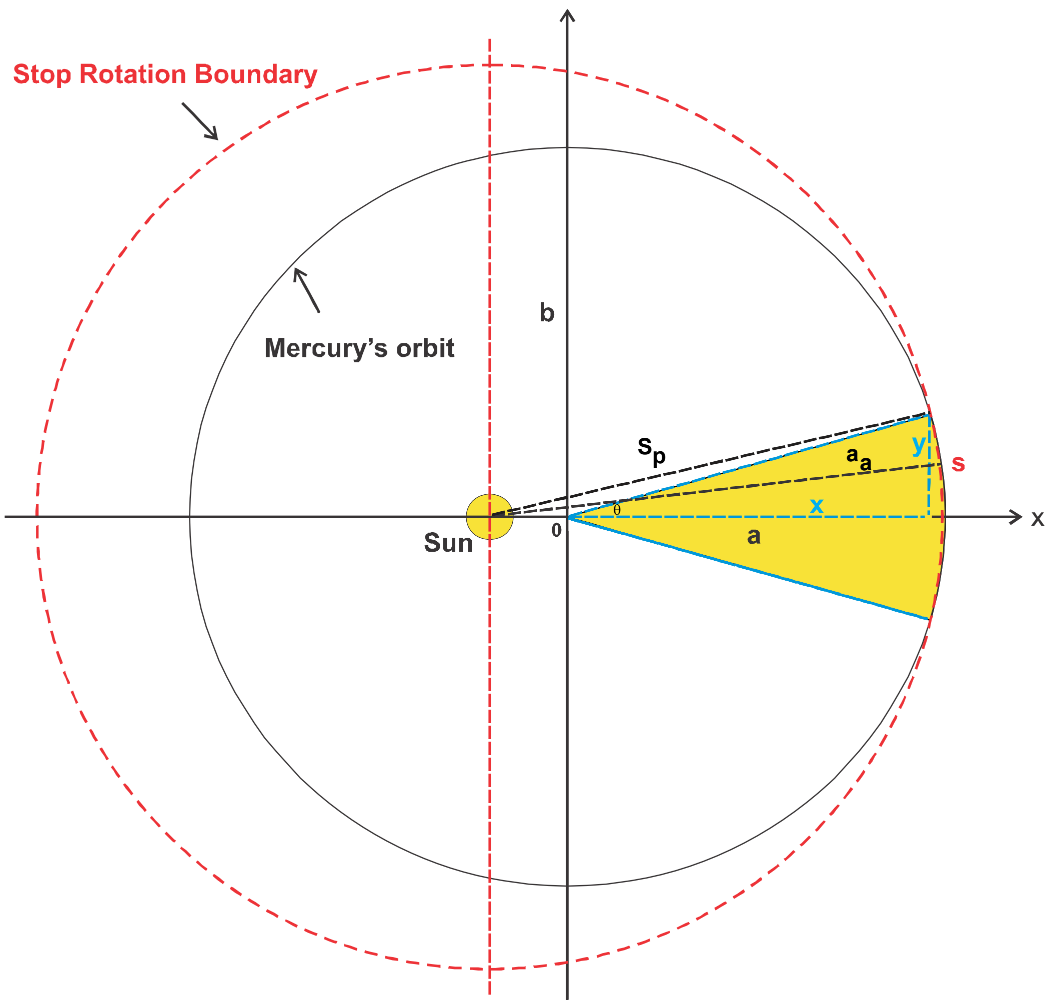
$s = 15770775.63$ Half of Mercury's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a\frac{\int_{\pi - \frac{s}{a}}^{\pi} (1 - e \cdot cos(E)) \cdot \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}{\int_{\pi - \frac{s}{a}}^{\pi} \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 69671322.46$ Mercury's average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^{\ 3}}{a^3}}$$

$n = 0.11562537$ Ratio of the Mercury's orbit out of S_p Boundary to the whole orbit



Mercury's orbit overlapping the Stop Rotation's domain

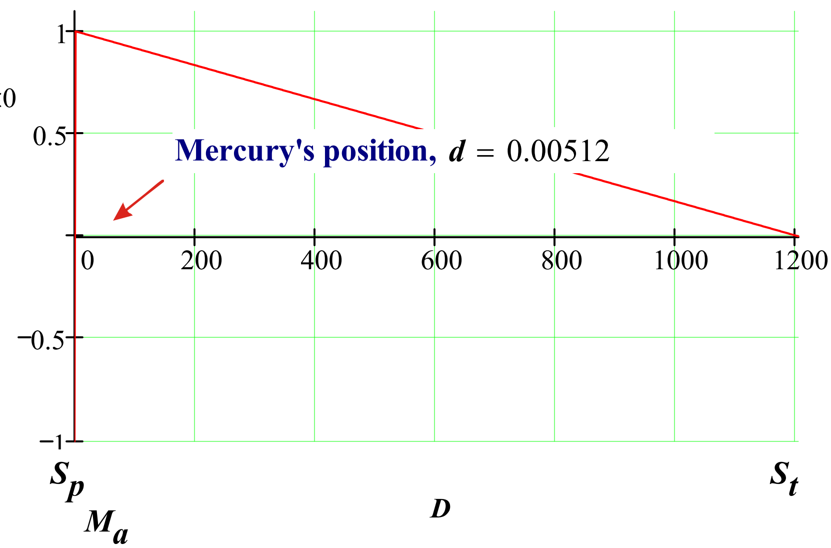
$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p}\right), \frac{a - S_p}{M_a - S_p}\right)$$

$d = 0.00511642$ Mercury's corresponding distance to the Sun relative to S_p on the X axis of the graph

Non-proportional Rotation Graph

Rotation

$$\begin{cases} -1-\frac{0.04}{D-(\sqrt{0.29}-0.5)}-(\sqrt{0.29}-0.5) & if\ D<0 \\ \frac{-0.04}{D+(\sqrt{0.29}-0.5)}+(0.5+\sqrt{0.29}) & if\ 0\le D\le 1 \\ \frac{-D+1}{\frac{S_t-M_a}{M_a-S_p}}+1 & if\ 1<D \end{cases}$$

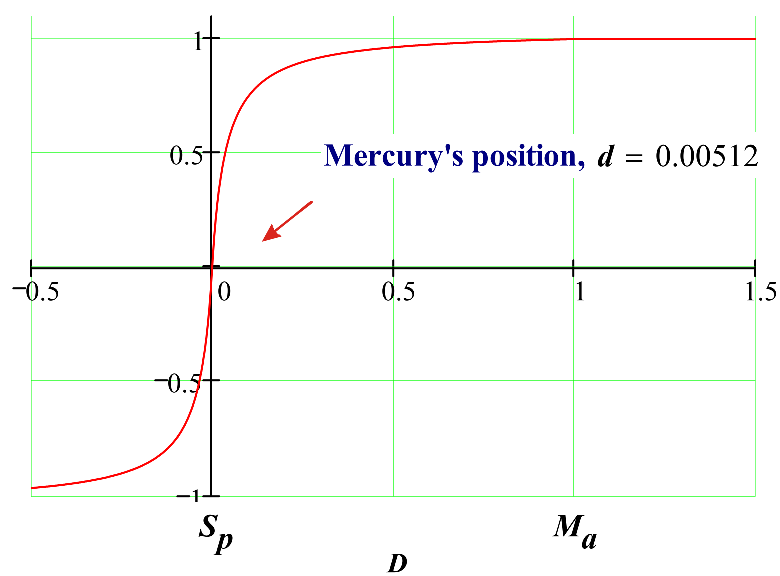


Distance

Left end of the Rotation Graph

Rotation

$$\begin{cases} -1-\frac{0.04}{D-(\sqrt{0.29}-0.5)}-(\sqrt{0.29}-0.5) & if\ D<0 \\ \frac{-0.04}{D+(\sqrt{0.29}-0.5)}+(0.5+\sqrt{0.29}) & if\ 0\le D\le 1 \\ \frac{-D+1}{\frac{S_t-M_a}{M_a-S_p}}+1 & if\ 1<D \end{cases}$$



Distance

$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.12177712$ Mercury's corresponding Influenced Rotation by the Sun on the Y axis of the graph

$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 0.00004891$ Mercury's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i := if\left[a > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot \left(\omega_{Mi} + \omega_F - t_r\right) \cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega i = 0.01721201$ **Mercury's end result Rotation (p.d.)**

Part 2

Mercury's Total Rotation

$$T := if\left(\omega i \le 0, 0, if\left(t \le 90, \frac{1}{\omega i}, \frac{-1}{\omega i}\right)\right)$$

$T = 58.099$ **Mercury's Sidereal Rotation Period (day)**

If (T = 0 , Mercury's Synchronous Tropical Rotation)

Observation

$T_o := 58.65$ **Mercury's Sidereal Rotation Period) (day)**

If (T = 0 , Mercury's Synchronous Tropical Rotation)

$$\%Diff := \frac{\left(T - T_o\right) \cdot 200}{T + T_o}$$

$\%Diff = -0.94395916$ Percentage deference between the calculation and the observation