Earth's Rotation Equations

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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Earth	Sun
Mass (kg)	$m := 5.9736 \cdot 10^{24}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho := 5.515$	$\rho_s := 1.408$
Axis Tilt (deg)	$t := 23.45$	$t_s := 7.25$
Semi-major Axis (km)	$a := 149600000$	
Orbit Eccentricity (deg)	$e := 0.01671022$	
Orbit Inclination (degree), with respect to equator	$i := 7.155$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$\omega_F = 0.48673034$ Earth's Free Rotation (per day)

Part 1

Earth's Influenced Rotation
by the influence of the Sun

$$q := a \cdot (1 - e)$$

$q = 147100151.1$ Earth's Perihelion Distance (km)

$$Q := a \cdot (1 + e)$$

$Q = 152099848.9$ Earth's Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99610642$ Earth's Influenced Rotation Reduction Factor by Orbit Inclination

$$\omega_{Mi} := \frac{\sqrt[6]{m \cdot i_r \div M \div \sqrt[6]{\rho}}}{i_M}$$

$\omega_{Mi} = 0.7108803$ Earth's Maximum Influenced Rotation by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{St}}$$

$S_t = 118594083398.6$ Earth's Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 210720920.9$ Earth's Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Sp}}$$

$S_p = 112321924.7$ Earth's Stop Rotation Distance to the Sun (km)

Calculating Earth's average distance to the Sun, if ($q < S_p < Q$)

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a}{e}, 0\right), 0\right)$$

$x = 0$ X value at Earth's orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 149579112$ Earth's Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 149579112.03$ Y value at the Earth's orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Earth's orbit out of S_p Boundary (rad)

$$s := a \cdot \int_0^{\theta} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

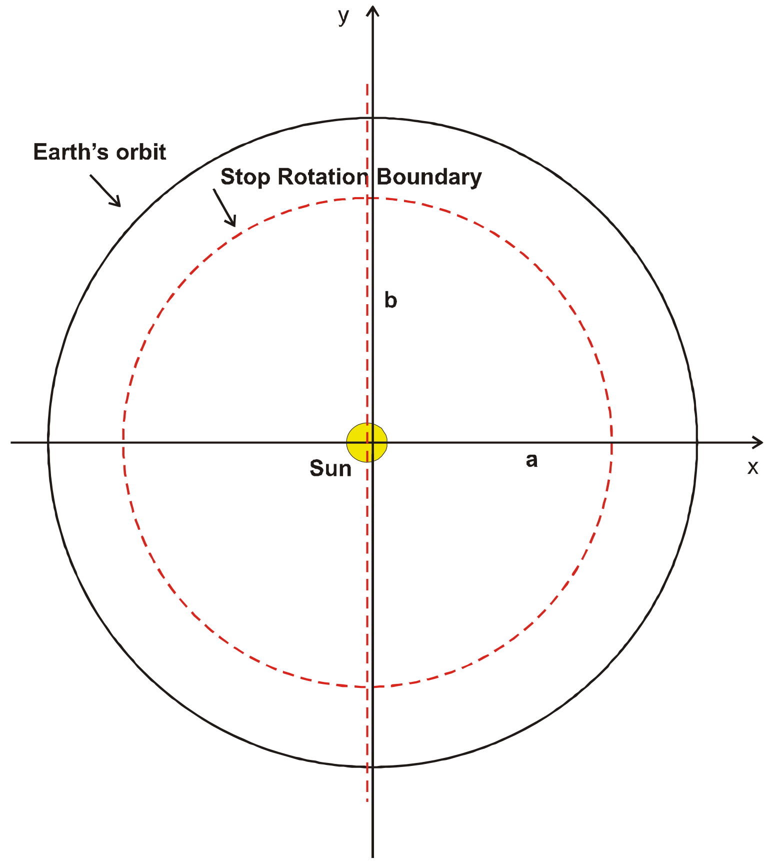
$s = 234974725.4$ Half of Earth's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a \frac{\int_{\pi - \frac{s}{a}}^{\pi} (1 - e \cdot cos(E)) \cdot \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}{\int_{\pi - \frac{s}{a}}^{\pi} \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Earth's average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a \cdot \int_0^{\pi} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta} \cdot \sqrt{\frac{a_a^{\ 3}}{a^3}}$$

$n = 0$ Ratio of the Earth's orbit out of S_p Boundary to the whole orbit

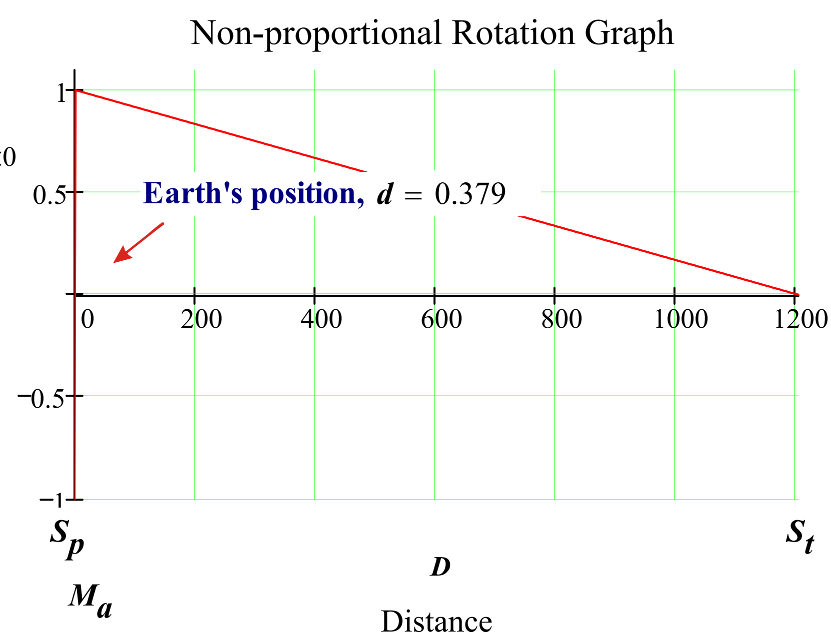


Earth's orbit relative to the Stop Rotation Boundary

$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p}\right), \frac{a - S_p}{M_a - S_p}\right)$$

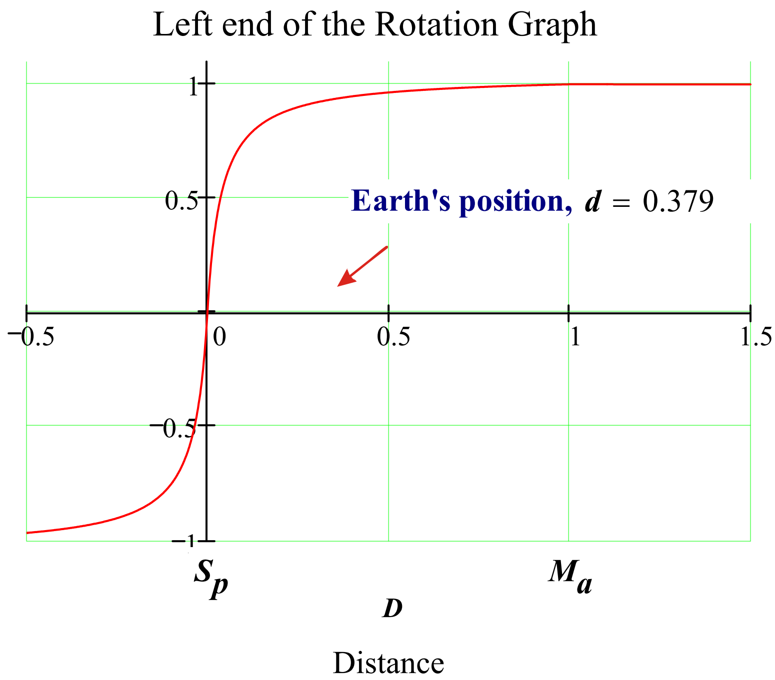
$d = 0.37884609$ Earth's corresponding distance to the Sun relative to S_p on the X axis of the graph

$$\text{Rotation} \; \left| \begin{array}{l} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \quad if \;\; D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \quad if \;\; 0 \le D \le 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 \quad if \;\; 1 < D \end{array} \right.$$



Rotation

$$\begin{cases} -1-\dfrac{0.04}{D-(\sqrt{0.29}-0.5)}-(\sqrt{0.29}-0.5) & \text{if } D<0 \\ \dfrac{-0.04}{D+(\sqrt{0.29}-0.5)}+(0.5+\sqrt{0.29}) & \text{if } 0\le D\le 1 \\ \dfrac{-D+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<D \end{cases}$$



$$\omega(d) := \begin{cases} -1-\dfrac{0.04}{d-(\sqrt{0.29}-0.5)}-(\sqrt{0.29}-0.5) & \text{if } d<0 \\ \dfrac{-0.04}{d+(\sqrt{0.29}-0.5)}+(0.5+\sqrt{0.29}) & \text{if } 0\le d\le 1 \\ \dfrac{-d+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<d \end{cases}$$

$\omega(d) = 0.94267655$ Earth's corresponding Influenced Rotation by the Sun on the Y axis of the graph

$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 0.12682029$ Earth's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_1 := if\left[a > M_a, \omega(d)\cdot\omega_{Mi} + \omega_F - t_r, \left[\omega(d)\cdot(\omega_{Mi} + \omega_F - t_r)\cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega i_1 = 1.00940895$ **Earth's end result Rotation (p.d.)**

Part 2

Earth's Influenced Rotation by the influence of the Moon

$if(q < S_t)$

Moon's Facts

$a_m := 384400$	Moon Semi-major Axis (km)
$e_m := 0.0549$	Moon Orbit Eccentricity (degree)
$i_m := 23.43$	Moon Orbit Inclination (degree)
$t_m := 6.68$	Moon Axis Tilt (degree)
$m_m := 7.349 \cdot 10^{22}$	Moon Mass (kg)

$q := a_m \cdot (1 - e_m)$

$q = 363296.4$ Moon's Perihelion Distance (km)

$Q := a_m \cdot (1 + e_m)$

$Q = 405503.6$ Moon's Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99660566$ Moon's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000546622$ Earth's Maximum Influenced Rotation by the Moon (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 91191397.4$ Earth's Start Influenced Rotation Distance to the Moon (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 162031.1$ Earth's Maximum Influenced Rotation Distance to the Moon (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

$S_p = 86368.5$ Earth's Stop Rotation Distance to the Moon (km)

Calculating Earth's average distance to the Moon, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a_m}{e_m}, 0\right), 0\right)$$

$x = 0$ X value at Earth's orbit intersection with S_p Boundary (km)

$$b := a_m \sqrt{1 - e_m^{\,2}}$$

$b = 383820.3$ Earth's Semi-minor Axis (km)

$$y := b\sqrt{a_m^{\,2} - x^2} \div a_m$$

$y = 383820.27$ Y value at Earth's orbital intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of Earth's orbit out of S_p Boundary (rad)

$$s := a_m \cdot \int_0^{\theta} \sqrt{1 - e_m^{\,2} \cdot sin(\theta)^2}\, d\theta$$

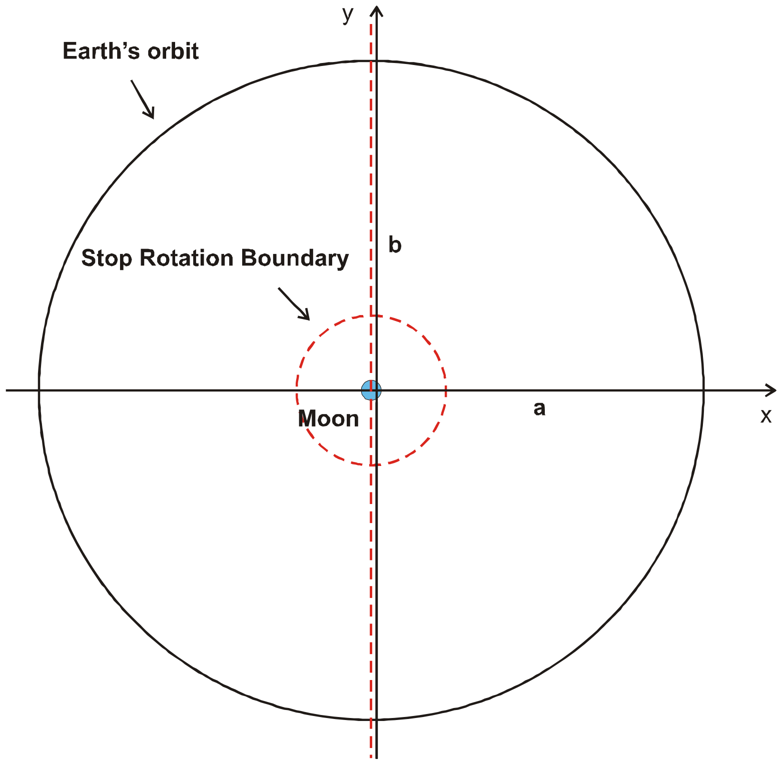
$s = 603358.88$ Half of Earth's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a_m \frac{\displaystyle\int_{\pi-(s\div a_m)}^{\pi} \frac{\sqrt{1 - e_m^{\,2} \cdot cos(E)^2}}{1 \div \left(1 - e_m \cdot cos(E)\right)}\, dE}{\displaystyle\int_{\pi-(s\div a_m)}^{\pi} \sqrt{1 - e_m^{\,2} \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Earth's average distance to the Moon outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a_m \cdot \displaystyle\int_0^{\pi} \sqrt{1 - e_m^{\,2} \cdot sin(\theta)^2}\, d\theta} \cdot \sqrt{\frac{a_a^{\,3}}{a^3}}$$

$n = 0$ Temporal ratio of the Earth's orbit out of S_p Boundary to the whole orbit



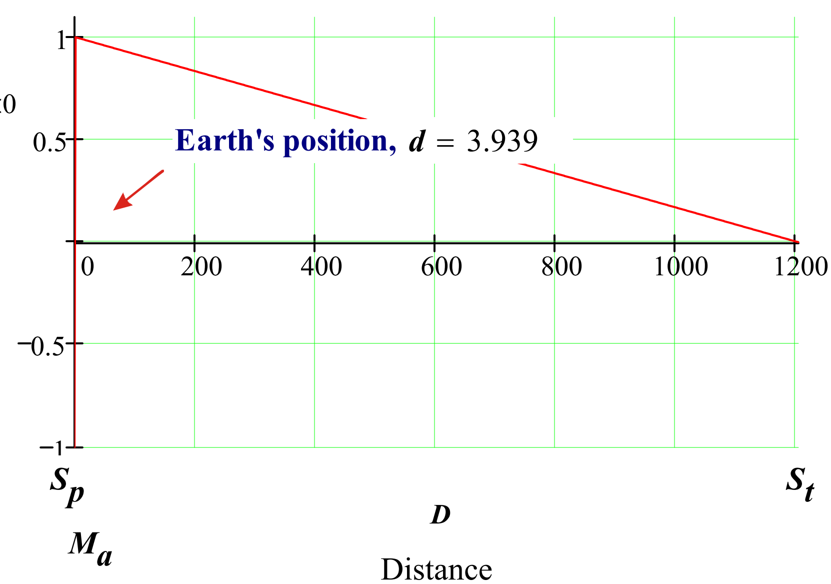
Earth/Moon orbit relative to the Stop Rotation Boundary

$$d := if\left(q < S_p, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p}\right)$$

$d = 3.93895165$ Earth's corresponding distance to the Moon relative to S_p on the X axis of the graph

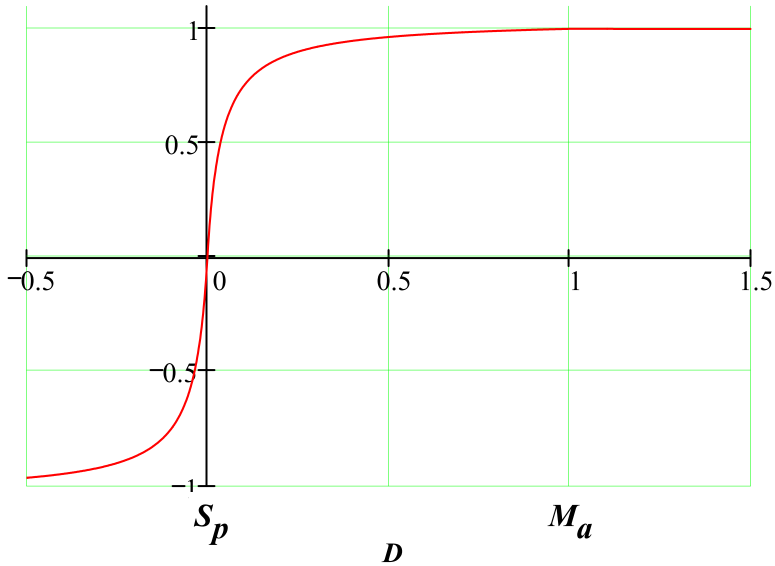
Non-proportional Rotation Graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \quad \begin{cases} -1-\dfrac{0.04}{D-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \textit{if} \;\; D<0 \\ \dfrac{-0.04}{D+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \textit{if} \;\; 0\le D\le 1 \\ \dfrac{-D+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \textit{if} \;\; 1<D \end{cases}$$

Left end of the Rotation Graph



Distance

$$\omega(d) := \begin{cases} -1\cdot\left(\dfrac{S_p}{M_a-S_p}\right)-\dfrac{0.04\cdot\left(\dfrac{S_p}{M_a-S_p}\right)}{d-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \textit{if} \;\; d<0 \\ \dfrac{-0.04}{d+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \textit{if} \;\; 0\le d\le 1 \\ \dfrac{-d+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \textit{if} \;\; 1<d \end{cases}$$

$\omega(d) = 0.99755717$ Earth's corresponding Influenced Rotation by the Moon on the Y axis of the graph

$$t_r := \textit{if}\left(a_m < M_a, \textit{if}\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), \textit{if}\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 1.4195638\times10^{-4}$ Earth's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_2 := \textit{if}\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi}-t_r\right), \textit{if}\left(q < S_t, \omega(d)\cdot\omega_{Mi}-t_r, 0\right)\right]$$

$\omega i_2 = 4.033306\times10^{-4}$ **Earth's end result Influenced Rotation by the Moon (p.d.)**
(Negative number means the reduction amount from Earth's Free Rotation)

Part 3

Earth's Total Rotation

$$\omega_s := \sum_{i=1}^{2} \omega i_i$$

$\omega_s = 1.00981228$ Earth's Total Rotation (p.d.)

$$T := if\left(\omega i_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s}\right)\right)$$

$T = 0.990283$ **Earth's Sidereal Rotation Period (day)**

If (T = 0 , Earth's Synchronous Tropical Rotation)

Observation

$T_o := 0.997271$ **Earth's Sidereal Rotation Period (day)**

If (T = 0 , Earth's Synchronous Tropical Rotation)

$$\%Diff := \frac{\left(T - T_o\right) \cdot 200}{T + T_o}$$

$\%Diff = -0.7032$ Percentage deference between the calculation and the observation