Mars' Rotation Equations

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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Mars	Sun
Mass (kg)	$m := 6.4185 \cdot 10^{23}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho := 3.94$	$\rho_s := 1.408$
Axis Tilt (deg)	$t := 25.19$	$t_s := 7.25$
Semi-major Axis (km)	$a := 227920000$	
Orbit Eccentricity (deg)	$e := 0.09341233$	
Orbit Inclination (degree), with respect to equator	$i := 5.66$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$\omega_F = 0.59666542$ Mars' Free Rotation (per day)

Part 1

Mars' Influenced Rotation by the influence of the Sun

$q := a \cdot (1 - e)$

$q = 206629461.7$ Mars' Perihelion Distance (km)

$Q := a \cdot (1 + e)$

$Q = 249210538.3$ Mars' Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99756233$ Mars' Influenced Rotation Reduction Factor by Orbit Inclination

$$\omega_{Mi} := \frac{\sqrt[6]{m \cdot i_r \div M \div \sqrt[6]{\rho}}}{i_M}$$

$\omega_{Mi} = 0.51853329$ Mars' Maximum Influenced Rotation by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{St}}$$

$S_t = 81790269932.4$ Mars' Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 145326988.6$ Mars' Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Sp}}$$

$S_p = 77464577.3$ Mars' Stop Rotation Distance to the Sun (km)

Calculating Mars' average distance to the Sun, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a}{e}, 0\right), 0\right)$$

$x = 0$ X value at Mars' orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 226923421.8$ Mars' Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 226923421.84$ Y value at the Mars' orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Mars' orbit out of S_p Boundary (rad)

$$s := a \cdot \int_0^{\theta} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

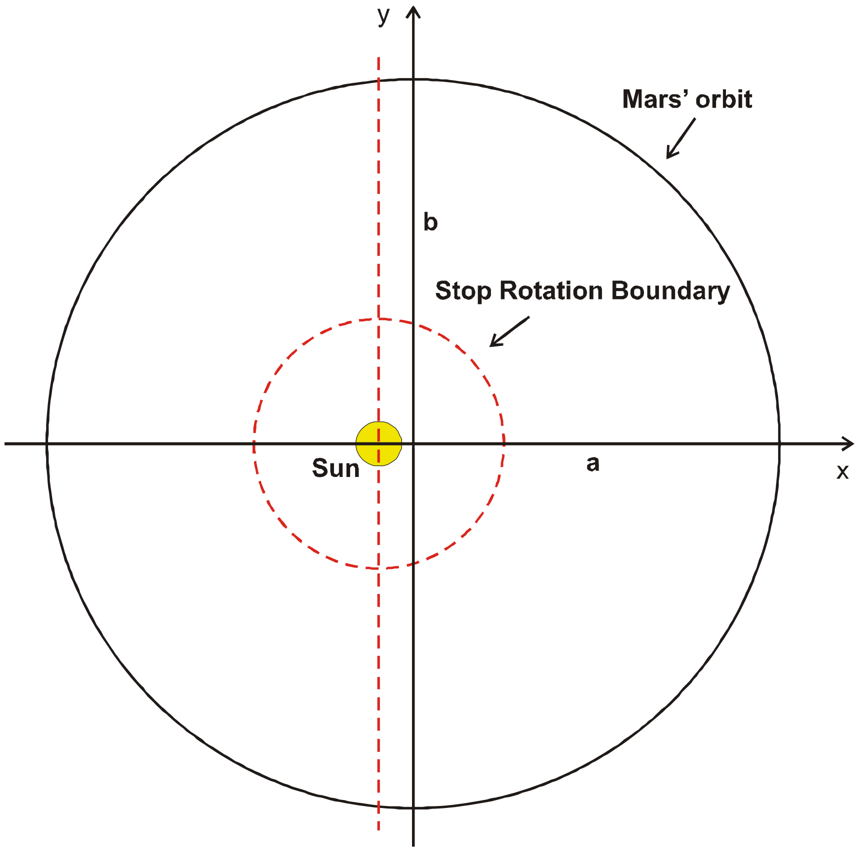
$s = 357233616.89$ Half of Mars' orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a\frac{\int_{\pi - \frac{s}{a}}^{\pi} (1 - e \cdot cos(E)) \cdot \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}{\int_{\pi - \frac{s}{a}}^{\pi} \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Mars' average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a \cdot \int_0^{\pi} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta} \cdot \sqrt{\frac{{a_a}^3}{a^3}}$$

$n = 0$ Ratio of the Mars' orbit out of S_p Boundary to the whole orbit



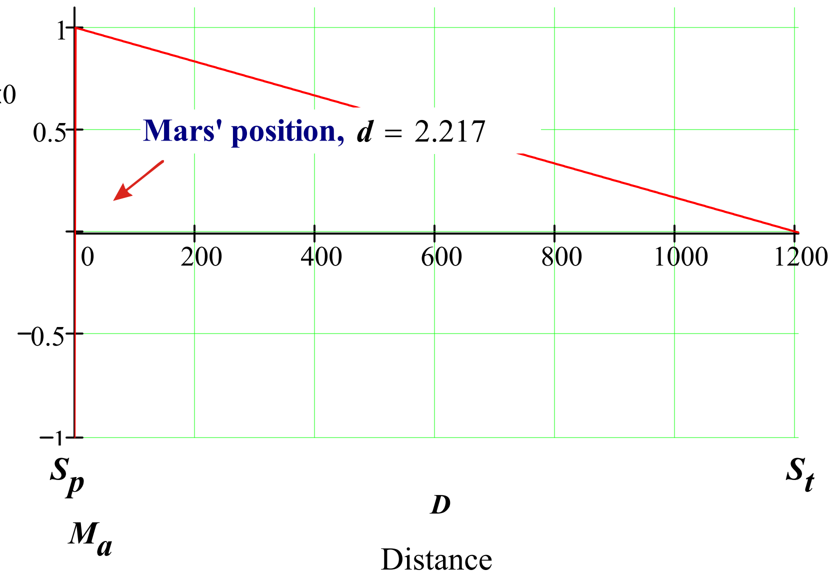
Mars’ orbit relative to the Stop Rotation Boundary

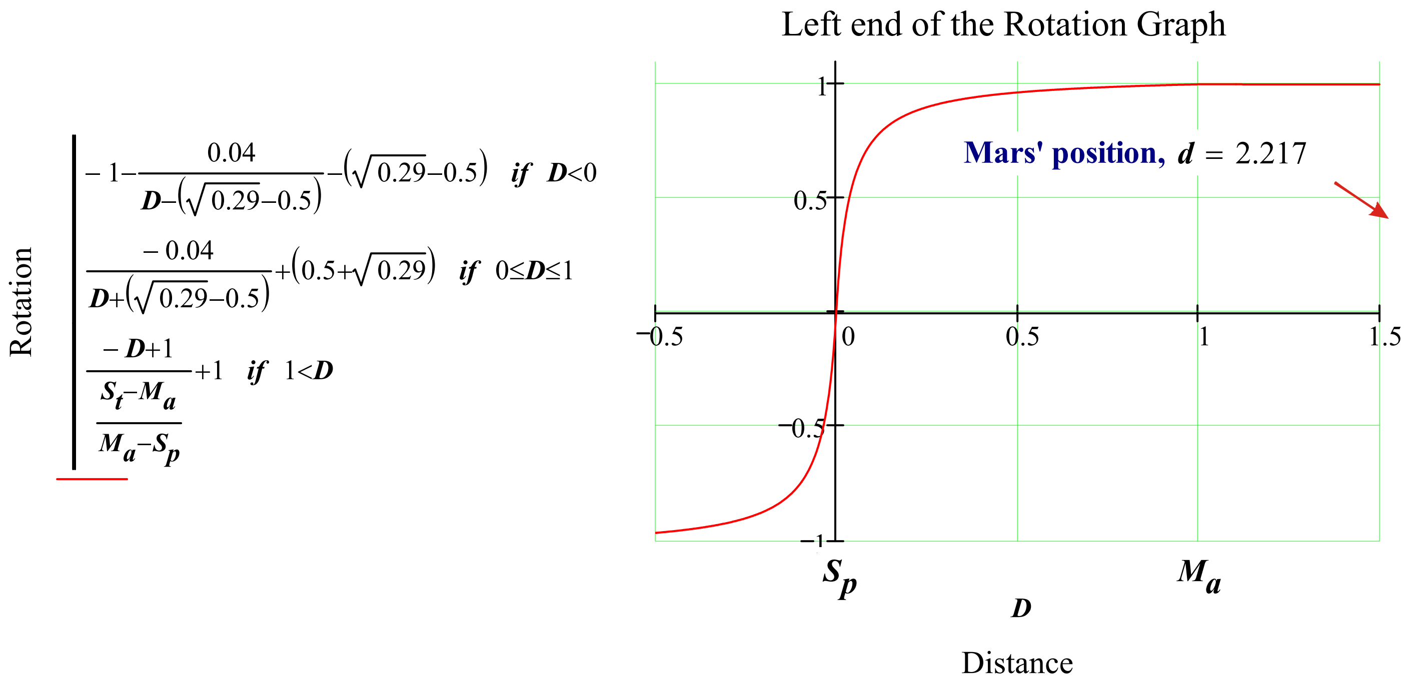
$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p}\right), \frac{a - S_p}{M_a - S_p}\right)$$

$d = 2.21706568$ Mars' corresponding distance to the Sun relative to S_p on the X axis of the graph

Non-proportional Rotation Graph

$$\text{Rotation} \quad \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$





$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.99898839$ Mars' corresponding Influenced Rotation by the Sun on the Y axis of the graph

$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 0.14498489$ Mars' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_1 := if\left[a > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot \left(\omega_{Mi} + \omega_F - t_r\right) \cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega i_1 = 0.96968927$ **Mars' end result Rotation (p.d.)**

Part 2

Mars' Influenced Rotation by the influence of the Phobos

$if\left(q < S_t\right)$

Phobos' Facts

$a_m := 9378$ Phobos Semi-major Axis (km)

$e_m := 0.0151$ Phobos Orbit Eccentricity (degree)

$i_m := 1.08$ Phobos Orbit Inclination (degree)

$t_m := 0.046$ Phobos Axis Tilt (degree)

$m_m := 1.06 \cdot 10^{16}$ Phobos Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 9236.4$ Phobos' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 9519.6$ Phobos' Aphelion Distance (km)

$$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 0.99999984$ Phobos' Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000000023$ Mars' Maximum Influenced Rotation by the Phobos (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 3639.6$ Mars' Start Influenced Rotation Distance to the Phobos (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 6.5$ Mars' Maximum Influenced Rotation Distance to the Phobos (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

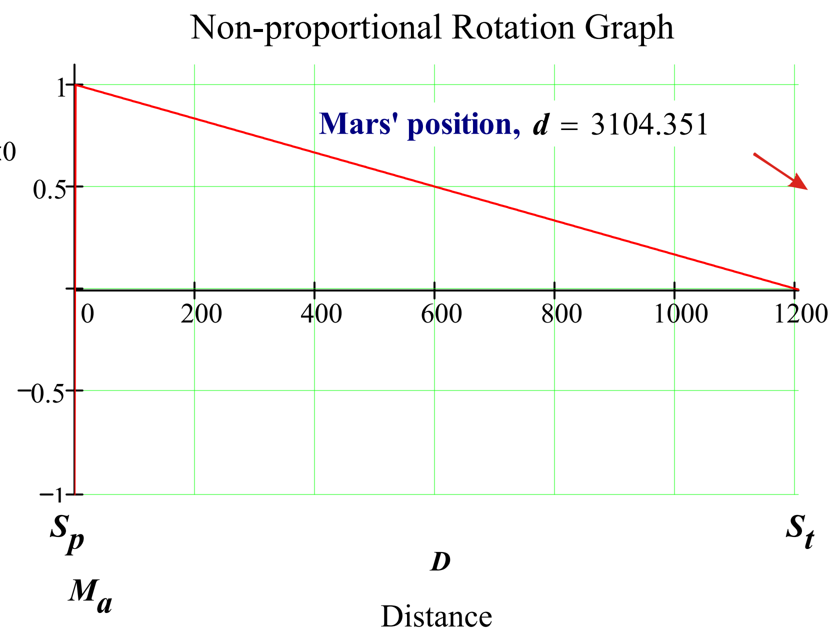
$S_p = 3.4$ Mars' Stop Rotation Distance to the Phobos (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 3.10435067 \times 10^3$ Mars' corresponding distance to the Phobos relative to S_p on the X axis of the graph

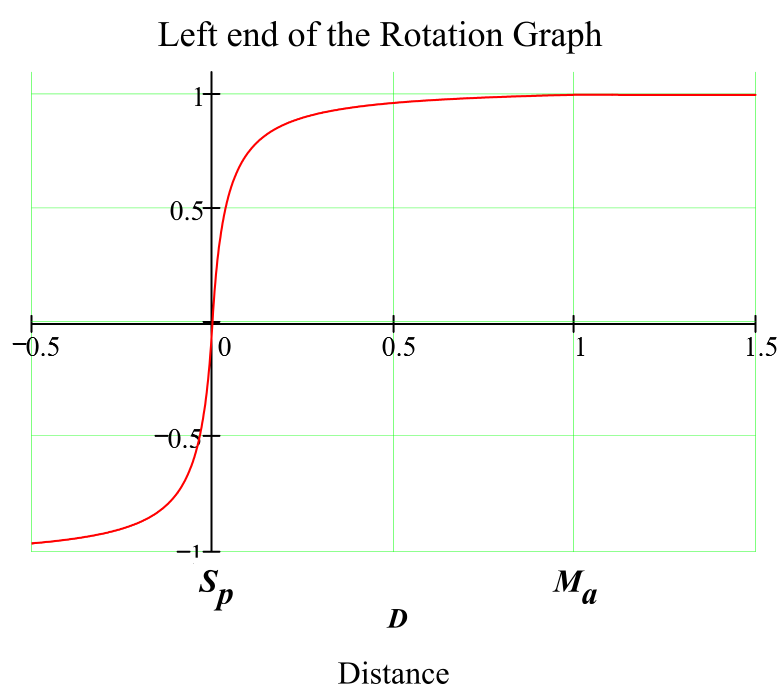
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\dfrac{S_p}{M_a - S_p}\right) - \dfrac{0.04\cdot\left(\dfrac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = -1.57947219$ Mars' corresponding Influenced Rotation by the Phobos on the Y axis of the graph

$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = -4.3734119 \times 10^{-10}$ Mars' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_2 := if\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_2 = 0$ **Mars' end result Influenced Rotation by the Phobos (p.d.)**

(Negative number means the reduction amount from Mars' Free Rotation)

Part 3

Mars' Influenced Rotation by the influence of the Deimos

Deimos' Facts

$a_m := 23459$ Deimos Semi-major Axis (km)

$e_m := 0.0002$ Deimos Orbit Eccentricity (degree)

$i_m := 1.79$ Deimos Orbit Inclination (degree)

$t_m := 0.897$ Deimos Axis Tilt (degree)

$m_m := 2.4 \cdot 10^{15}$ Deimos Mass (kg)

$$q := a_m \cdot (1 - e_m)$$

$q = 23454.3$ Deimos' Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$Q = 23463.7$ Deimos' Aphelion Distance (km)

$$i_r := \left(\left| cos\left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99993873$ Deimos' Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000000009$ Mars' Maximum Influenced Rotation by the Deimos (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 1352$ Mars' Start Influenced Rotation Distance to the Deimos (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 2.4$ Mars' Maximum Influenced Rotation Distance to the Deimos (km)

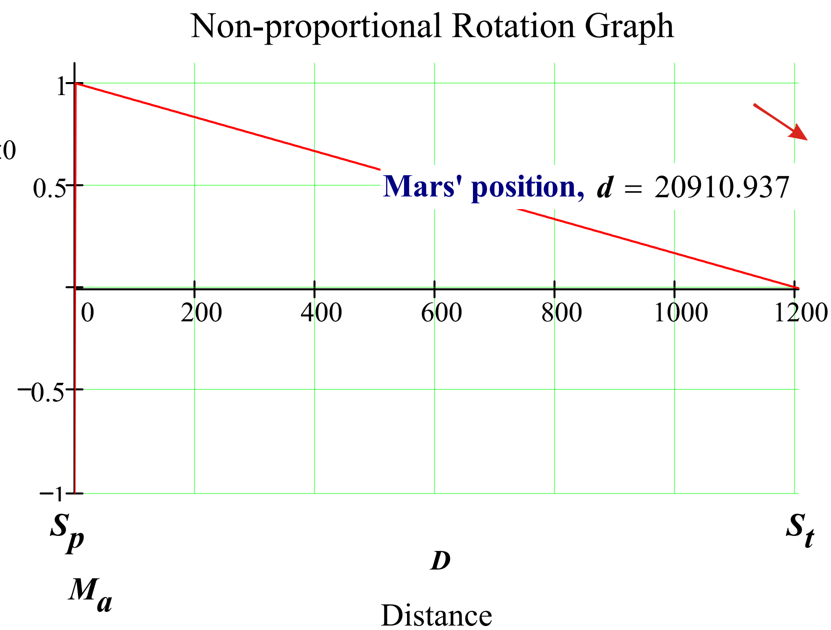
$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

$S_p = 1.3$ Mars' Stop Rotation Distance to the Deimos (km)

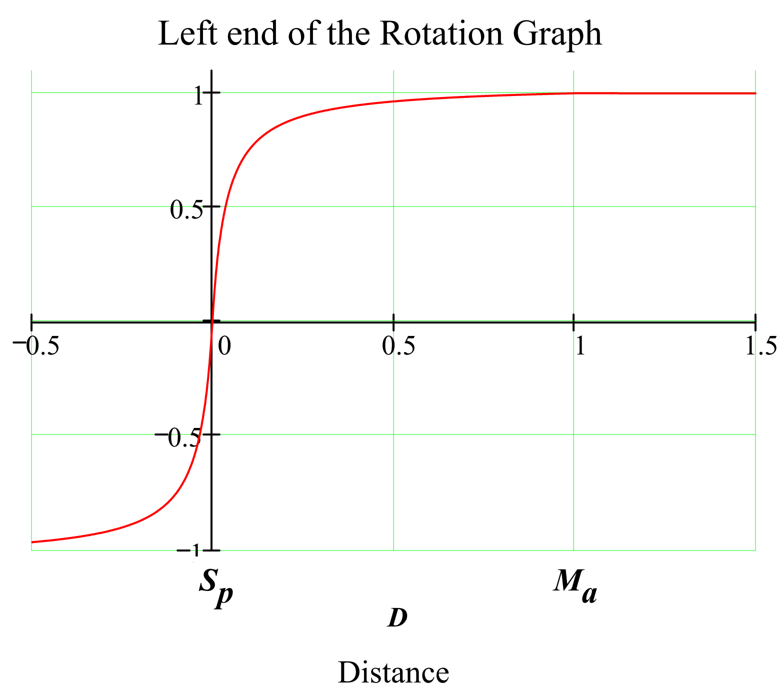
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 2.09109375 \times 10^4$ Mars' corresponding distance to the Deimos relative to S_p on the X axis of the graph

$$\text{Rotation} \quad \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \quad \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\dfrac{S_p}{M_a - S_p}\right) - \dfrac{0.04\cdot\left(\dfrac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = -16.38011843$ Mars' corresponding Influenced Rotation by the Deimos on the Y axis of the graph

$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = -2.7924616 \times 10^{-9}$ Mars' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_3 := if\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_3 = 0$ **Mars' end result Influenced Rotation by the Deimos (p.d.)**
(Negative number means the reduction amount from Mars' Free Rotation)

Part 4

Mars' Total Rotation

$$\omega_s := \sum_{i=1}^{3} \omega i_i$$

$\omega_s = 0.96968927$ Mars' Total Rotation (p.d.)

$$T := if\left(\omega i_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s}\right)\right)$$

$T = 1.031258$ **Mars' Sidereal Rotation Period (day)**

If (T = 0 , Mars' Synchronous Tropical Rotation)

Observation

$T_o := 1.025954$ **Mars' Sidereal Rotation Period (day)**

If (*T* = 0 , Mars' Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$\%Diff = 0.5157$ Percentage deference between the calculation and the observation