Saturn's Rotation Equations

Peyman Parsa

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www.peymanparsa.com

Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Saturn	Sun
Mass (kg)	$m := 5.6846 \cdot 10^{26}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho := 0.687$	$\rho_S := 1.408$
Axis Tilt (deg)	$t := 26.73$	$t_S := 7.25$
Semi-major Axis (km)	$a := 1433530000$	
Orbit Eccentricity (deg)	$e := 0.0541506$	
Orbit Inclination (degree), with respect to equator	$i := 5.51$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$\omega_F = 0.08039889$ Saturn's Free Rotation (per day)

Part 1

Saturn's Influenced Rotation by the influence of the Sun

$q := a \cdot (1 - e)$

$q = 1355903490.4$ Saturn's Perihelion Distance (km)

$Q := a \cdot (1 + e)$

$Q = 1511156509.6$ Saturn's Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99768973$ Saturn's Influenced Rotation Reduction Factor by Orbit Inclination

$$\omega_{Mi} := \frac{\sqrt[6]{m \cdot i_r \div M} \div \sqrt[6]{\rho}}{i_M}$$

$\omega_{Mi} = 2.1499096$ Saturn's Maximum Influenced Rotation by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{St}}$$

$S_t = 253467303966$ Saturn's Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 450367018.2$ Saturn's Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Sp}}$$

$S_p = 240062021.7$ Saturn's Stop Rotation Distance to the Sun (km)

Calculating Saturn's average distance to the Sun, if ($q < S_p < Q$)

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a}{e}, 0\right), 0\right)$$

$x = 0$ X value at Saturn's orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 1431426696$ Saturn's Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 1431426695.96$ Y value at the Saturn's orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Saturn's orbit out of S_p Boundary (rad)

$$s := a \cdot \int_0^{\theta} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

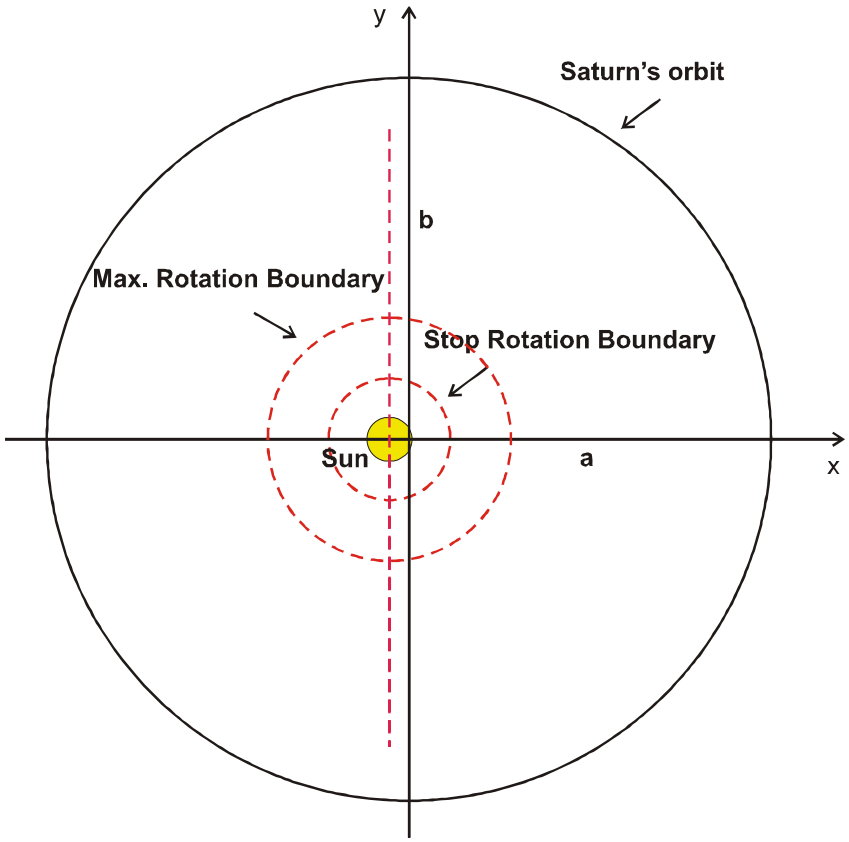
$s = 2250132030.41$ Half of Saturn's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a\frac{\int_{\pi - \frac{s}{a}}^{\pi} (1 - e \cdot cos(E)) \cdot \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}{\int_{\pi - \frac{s}{a}}^{\pi} \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Saturn's average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a \cdot \int_0^{\pi} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$n = 0$ Ratio of the Saturn's orbit out of S_p Boundary to the whole orbit

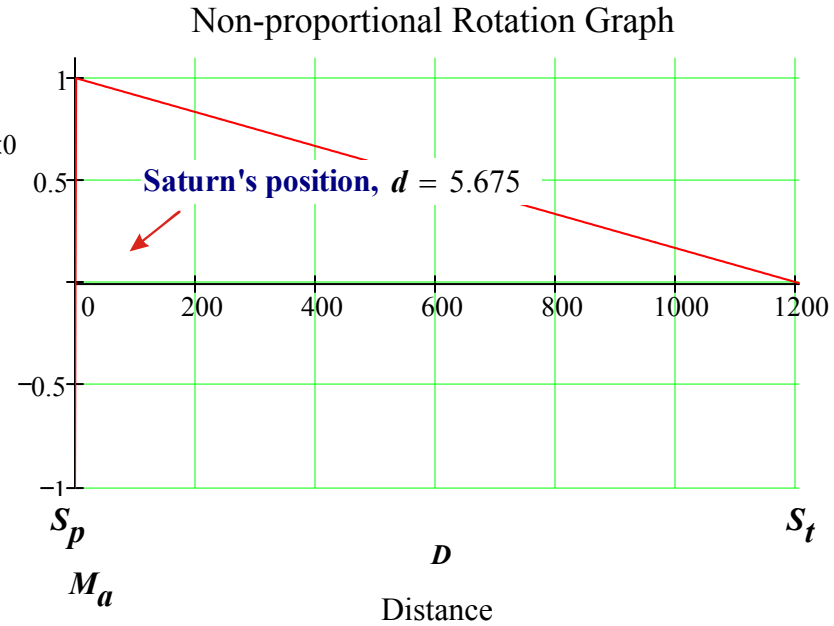


Saturn's orbit relative to the Stop and Max. Rotation Boundary

$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p}\right), \frac{a - S_p}{M_a - S_p}\right)$$

$d = 5.67493877$ Saturn's corresponding distance to the Sun relative to S_p on the X axis of the graph

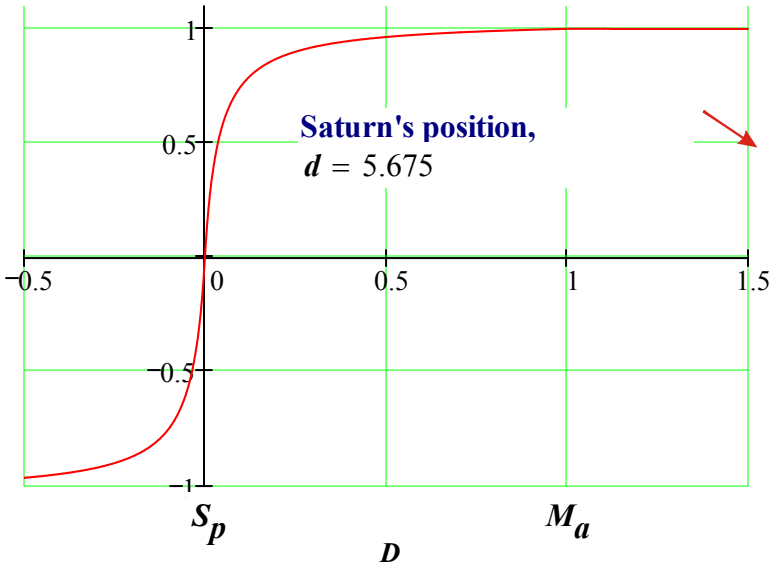
$$\text{Rotation} := \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1-\dfrac{0.04}{D-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } D<0 \\ \dfrac{-0.04}{D+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le D\le 1 \\ \dfrac{-D+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<D \end{cases}$$

Left end of the Rotation Graph



Distance

$$\omega(d) := \begin{cases} -1-\dfrac{0.04}{d-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } d<0 \\ \dfrac{-0.04}{d+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le d\le 1 \\ \dfrac{-d+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<d \end{cases}$$

$\omega(d) = 0.99611424$ Saturn's corresponding Influenced Rotation by the Sun on the Y axis of the graph

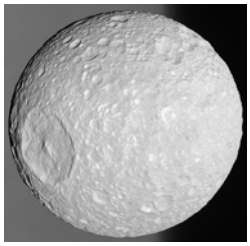
$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 0.02387847$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_1 := if\left[a > M_a, \omega(d)\cdot\omega_{Mi}+\omega_F-t_r, \left[\omega(d)\cdot\left(\omega_{Mi}+\omega_F-t_r\right)\cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega i_1 = 2.19807599$ **Saturn's end result Rotation (p.d.)**

Part 2



Saturn's Influenced Rotation
by the influence of the Mimas

$$if\left(q < S_t\right)$$

Mimas' Facts

$a_m := 185520$ Mimas Semi-major Axis (km)

$e_m := 0.0206$ Mimas Orbit Eccentricity (degree)

$i_m := 1.566$ Mimas Orbit Inclination (degree)

$t_m := 0.005$ Mimas Axis Tilt (degree)

$m_m := 3.79 \cdot 10^{19}$ Mimas Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 181698.3$ Mimas' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 189341.7$ Mimas' Aphelion Distance (km)

$$i_r := \left(\left|\cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 1$ Mimas' Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000002329$ Saturn's Maximum Influenced Rotation by the Mimas (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 274618.6$ Saturn's Start Influenced Rotation Distance to the Mimas (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 487.9$ Saturn's Maximum Influenced Rotation Distance to the Mimas (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

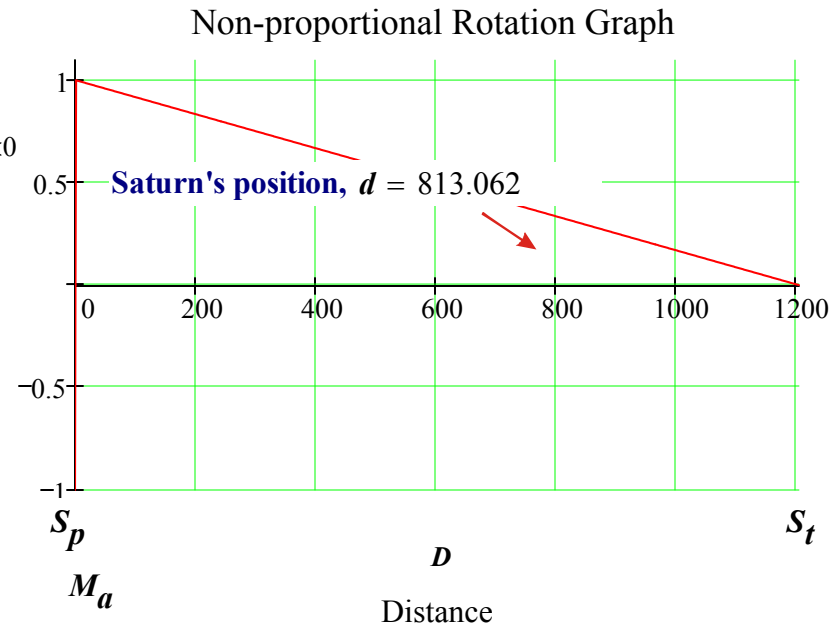
$S_p = 260.1$ Saturn's Stop Rotation Distance to the Mimas (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 813.06225426$ Saturn's corresponding distance to the Mimas relative to S_p on the X axis of the graph

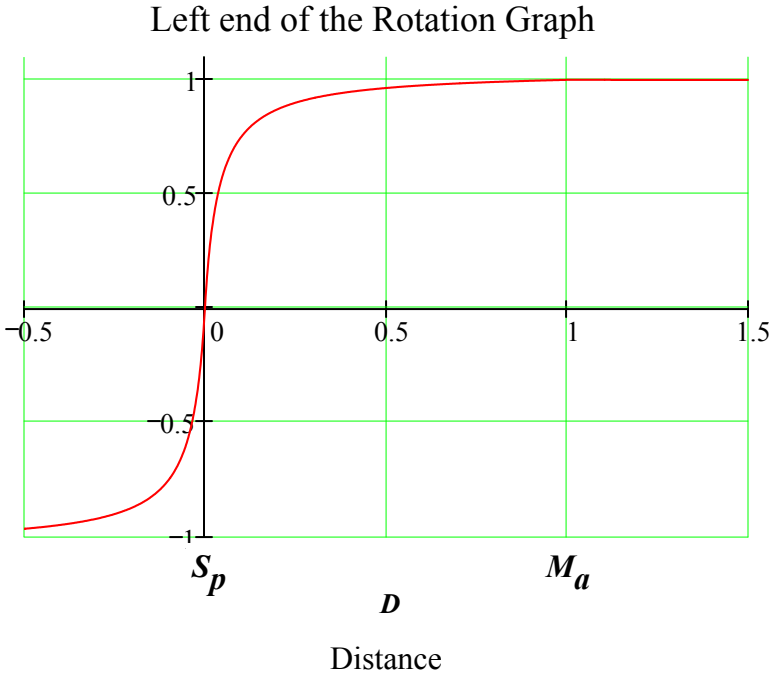
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04\cdot\left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.32502246$ Saturn's corresponding Influenced Rotation by the Mimas on the Y axis of the graph

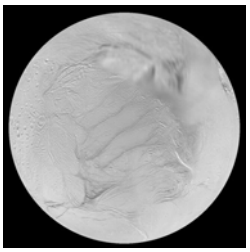
$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 1.3173187\times10^{-8}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_2 := if\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_2 = 7.4390654\times10^{-7}$ **Saturn's end result Influenced Rotation by the Mimas (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 3



Saturn's Influenced Rotation
by the influence of the Enceladus

$if\left(q < S_t\right)$

Enceladus' Facts

$a_m := 238020$ Enceladus Semi-major Axis (km)

$e_m := 0.0001$ Enceladus Orbit Eccentricity (degree)

$i_m := 0.010$ Enceladus Orbit Inclination (degree)

$t_m := 0.00$ Enceladus Axis Tilt (degree)

$m_m := 1.08 \cdot 10^{20}$ Enceladus Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 237996.2$ Enceladus' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 238043.8$ Enceladus' Aphelion Distance (km)

$$i_r := \left(\left|\cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 1$ Enceladus' Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000004682$ Saturn's Maximum Influenced Rotation by the Enceladus (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 551975.4$ Saturn's Start Influenced Rotation Distance to the Enceladus (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 980.8$ Saturn's Maximum Influenced Rotation Distance to the Enceladus (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

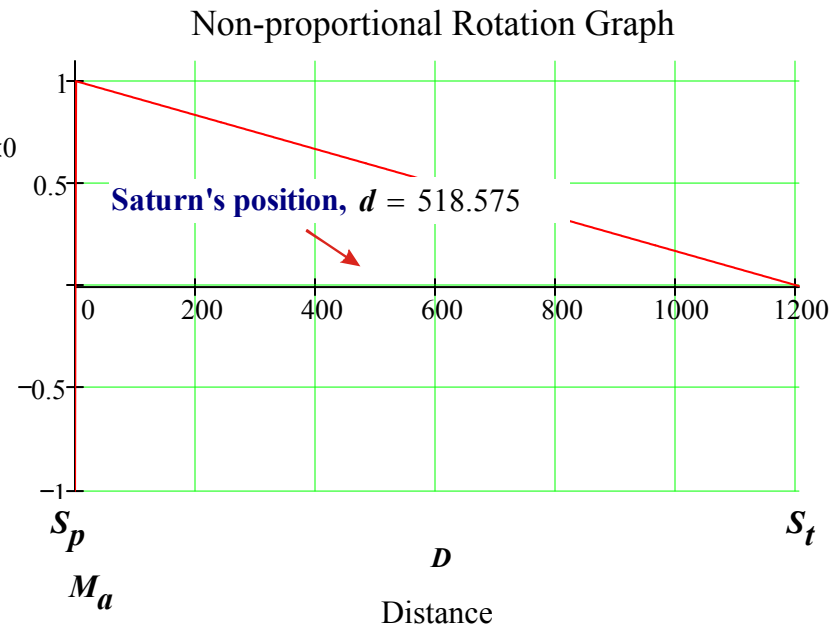
$S_p = 522.8$ Saturn's Stop Rotation Distance to the Enceladus (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 518.57453953$ Saturn's corresponding distance to the Enceladus relative to S_p on the X axis of the graph

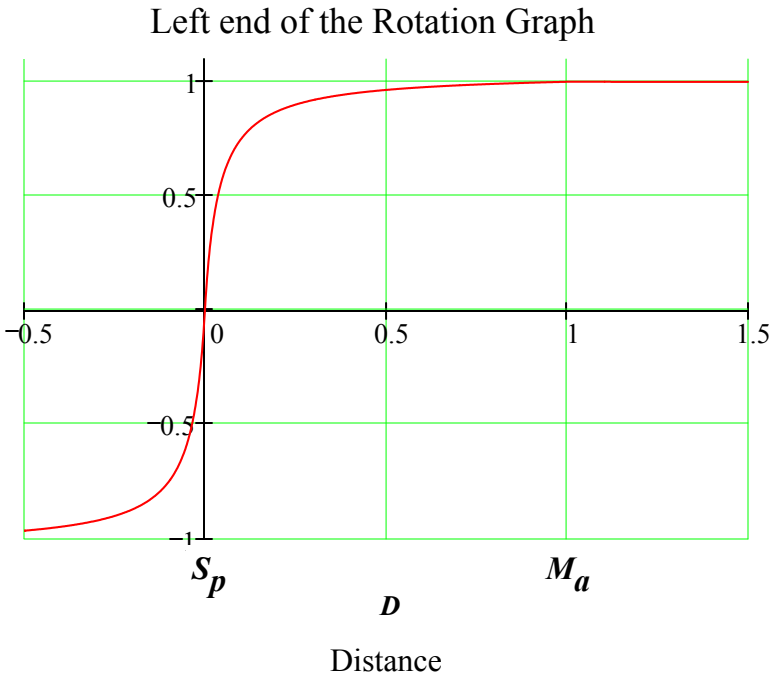
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.56979753$ Saturn's corresponding Influenced Rotation by the Enceladus on the Y axis of the graph

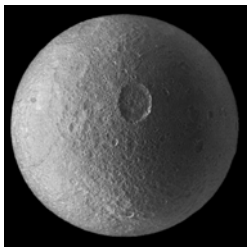
$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 2.9641215 \times 10^{-10}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_3 := if\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_3 = 2.6674129 \times 10^{-6}$ **Saturn's end result Influenced Rotation by the Enceladus (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 4



Saturn's Influenced Rotation by the influence of the Tethys

$if\left(q < S_t\right)$

Tethys' Facts

$a_m := 294700$ Tethys Semi-major Axis (km)

$e_m := 0.0001$ Tethys Orbit Eccentricity (degree)

$i_m := 0.168$ Tethys Orbit Inclination (degree)

$t_m := 0.034$ Tethys Axis Tilt (degree)

$m_m := 6.22 \cdot 10^{20}$ Tethys Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 294670.5$ Tethys' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 294729.5$ Tethys' Aphelion Distance (km)

$i_r := \left(\left|\cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.99999991$ Tethys' Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000015043$ Saturn's Maximum Influenced Rotation by the Tethys (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 1773498.3$ Saturn's Start Influenced Rotation Distance to the Tethys (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 3151.2$ Saturn's Maximum Influenced Rotation Distance to the Tethys (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

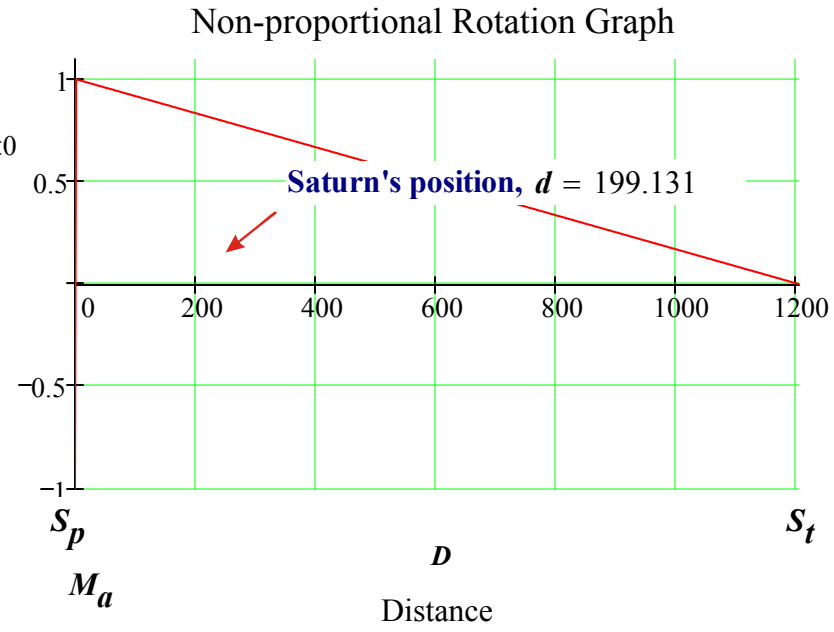
$S_p = 1679.7$ Saturn's Stop Rotation Distance to the Tethys (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 199.13119069$ Saturn's corresponding distance to the Tethys relative to S_p on the X axis of the graph

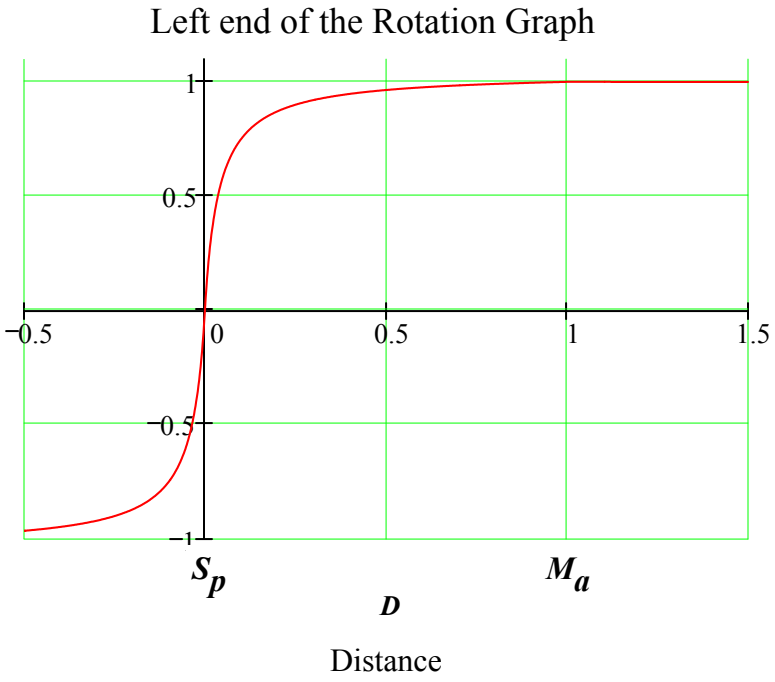
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.83531545$ Saturn's corresponding Influenced Rotation by the Tethys on the Y axis of the graph

$$t_r := \text{if}\left(a_m < M_a, \text{if}\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), \text{if}\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 2.3455587 \times 10^{-8}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_4 := \text{if}\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), \text{if}\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_4 = 1.2542038 \times 10^{-5}$ **Saturn's end result Influenced Rotation by the Tethys (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 5

Saturn's Influenced Rotation by the influence of the Dione

$if\left(q < S_t\right)$

Dione's Facts

$a_m := 377400$ Dione's Semi-major Axis (km)

$e_m := 0.0002$ Dione's Orbit Eccentricity (degree)

$i_m := 0.002$ Dione's Orbit Inclination (degree)

$t_m := 0.006$ Dione's Axis Tilt (degree)

$m_m := 1.1 \cdot 10^{21}$ Dione's Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 377324.5$ Dione's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 377475.5$ Dione's Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 1$ Dione's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000021999$ Saturn's Maximum Influenced Rotation by the Dione's (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 2593576$ Saturn's Start Influenced Rotation Distance to the Dione's (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 4608.3$ Saturn's Maximum Influenced Rotation Distance to the Dione's (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

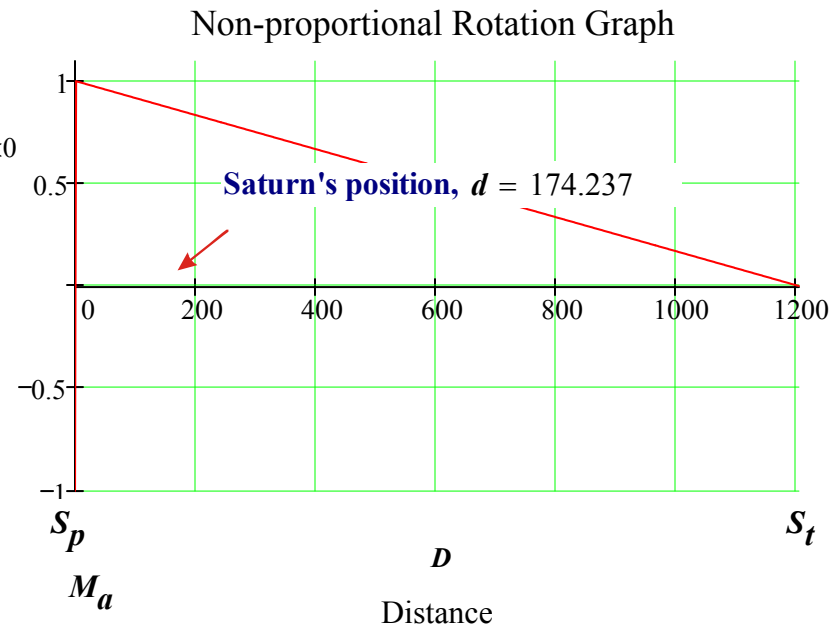
$S_p = 2456.4$ Saturn's Stop Rotation Distance to the Dione's (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 174.23657056$ Saturn's corresponding distance to the Dione's relative to S_p on the X axis of the graph

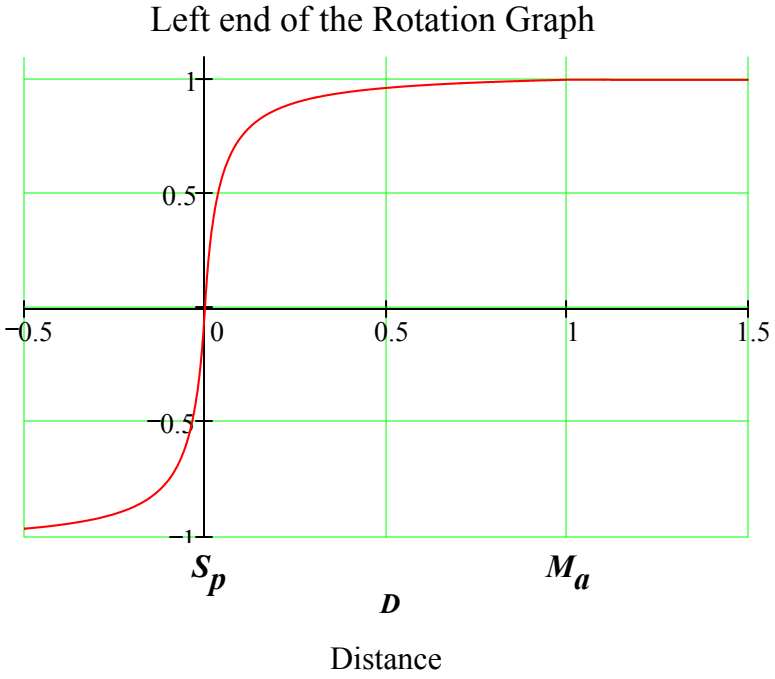
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04\cdot\left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.8560076$ Saturn's corresponding Influenced Rotation by the Dione's on the Y axis of the graph

$$t_r := \text{if}\left(a_m < M_a, \text{if}\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), \text{if}\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 4.1846808 \times 10^{-10}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_5 := \text{if}\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), \text{if}\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_5 = 1.8830645 \times 10^{-5}$ **Saturn's end result Influenced Rotation by the Dione (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 6

Saturn's Influenced Rotation by the influence of the Rhea

$$if\left(q < S_t\right)$$

Rhea's Facts

$a_m := 527100$ Rhea Semi-major Axis (km)

$e_m := 0.0009$ Rhea Orbit Eccentricity (degree)

$i_m := 0.327$ Rhea Orbit Inclination (degree)

$t_m := 0.029$ Rhea Axis Tilt (degree)

$m_m := 2.31 \cdot 10^{21}$ Rhea Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 526625.6$ Rhea's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 527574.4$ Rhea's Aphelion Distance (km)

$$i_r := \left(\left|\cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 0.99999994$ Rhea's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000036075$ Saturn's Maximum Influenced Rotation by the Rhea (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 4253161.1$ Saturn's Start Influenced Rotation Distance to the Rhea (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 7557.1$ Saturn's Maximum Influenced Rotation Distance to the Rhea (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

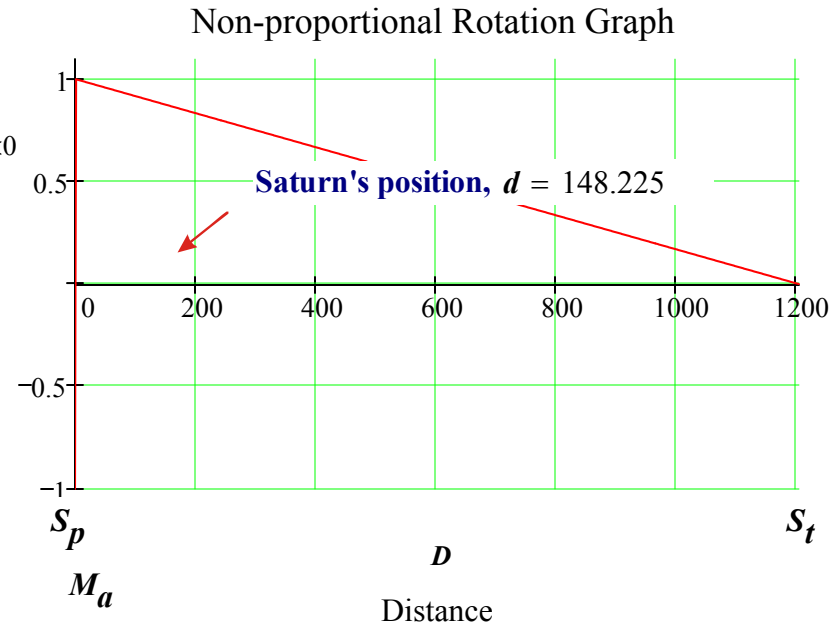
$S_p = 4028.2$ Saturn's Stop Rotation Distance to the Rhea (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 148.22511942$ Saturn's corresponding distance to the Rhea relative to S_p on the X axis of the graph

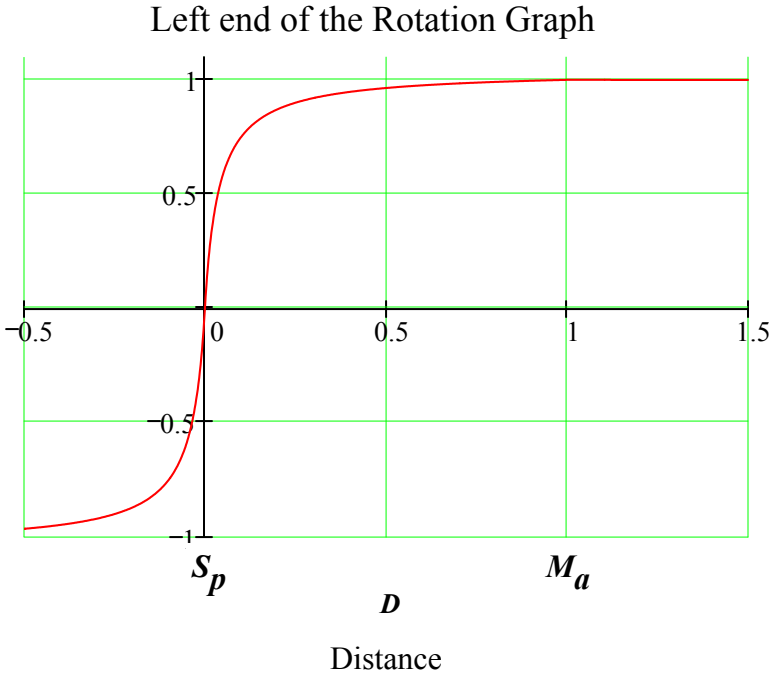
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.87762804$ Saturn's corresponding Influenced Rotation by the Rhea on the Y axis of the graph

$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

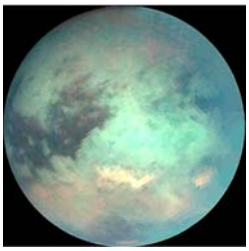
$t_r = 1.1503389 \times 10^{-7}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_6 := if\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_6 = 3.1545671 \times 10^{-5}$ **Saturn's end result Influenced Rotation by the Rhea (p.d.)**

(Negative number means the reduction amount from Saturn's Free Rotation)

Part 7



Saturn's Influenced Rotation by the influence of the Titan

$if\left(q < S_t\right)$

Titan's Facts

$a_m := 1221900$ Titan Semi-major Axis (km)

$e_m := 0.0288$ Titan Orbit Eccentricity (degree)

$i_m := 1.634$ Titan Orbit Inclination (degree)

$t_m := 1.942$ Titan Axis Tilt (degree)

$m_m := 1.3455 \cdot 10^{23}$ Titan Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 1186709.3$ Titan's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 1257090.7$ Titan's Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.99971282$ Titan's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000542047$ Saturn's Maximum Influenced Rotation by the Titan (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 63905592.3$ Saturn's Start Influenced Rotation Distance to the Titan (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 113549$ Saturn's Maximum Influenced Rotation Distance to the Titan (km)

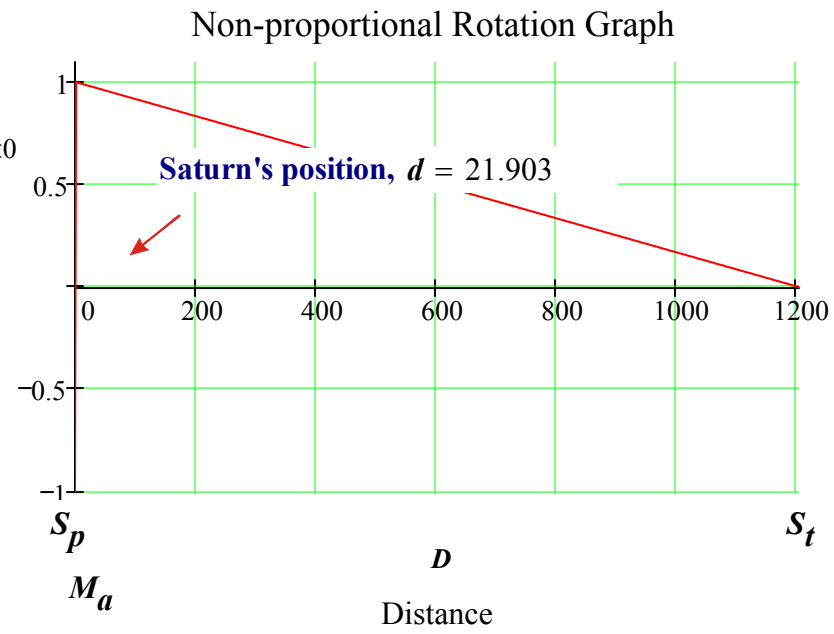
$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

$S_p = 60525.8$ Saturn's Stop Rotation Distance to the Titan (km)

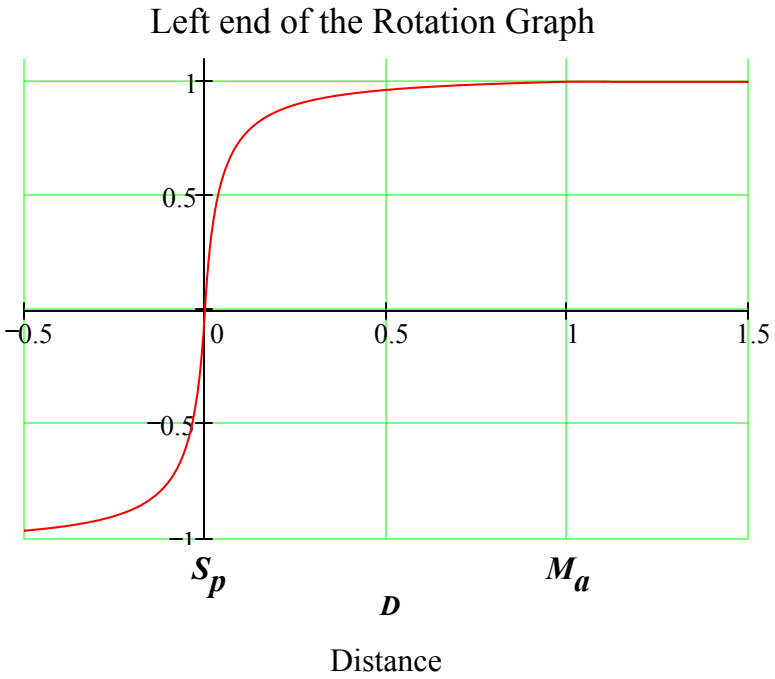
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 21.90310436$ Saturn's corresponding distance to the Titan relative to S_p on the X axis of the graph

$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04\cdot\left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.98262556$ Saturn's corresponding Influenced Rotation by the Titan on the Y axis of the graph

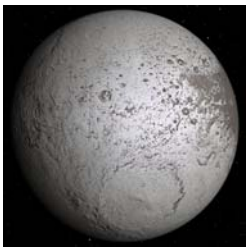
$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 9.6701836\times10^{-6}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_7 := if\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_7 = 5.2295927\times10^{-4}$ **Saturn's end result Influenced Rotation by the Titan (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 8



Saturn's Influenced Rotation by the influence of the Iapetus

$if\left(q < S_t\right)$

Iapetus's Facts

$a_m := 3560800$ Iapetus Semi-major Axis (km)

$e_m := 0.0284$ Iapetus Orbit Eccentricity (degree)

$i_m := 7.57$ Iapetus Orbit Inclination (degree)

$t_m := 14.84$ Iapetus Axis Tilt (degree)

$m_m := 1.81 \cdot 10^{21}$ Iapetus Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 3459673.3$ Iapetus' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 3661926.7$ Iapetus' Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.98332241$ Iapetus' Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000030575$ Saturn's Maximum Influenced Rotation by the Iapetus (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 3604725.7$ Saturn's Start Influenced Rotation Distance to the Iapetus (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 6405$ Saturn's Maximum Influenced Rotation Distance to the Iapetus (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

$S_p = 3414.1$ Saturn's Stop Rotation Distance to the Iapetus (km)

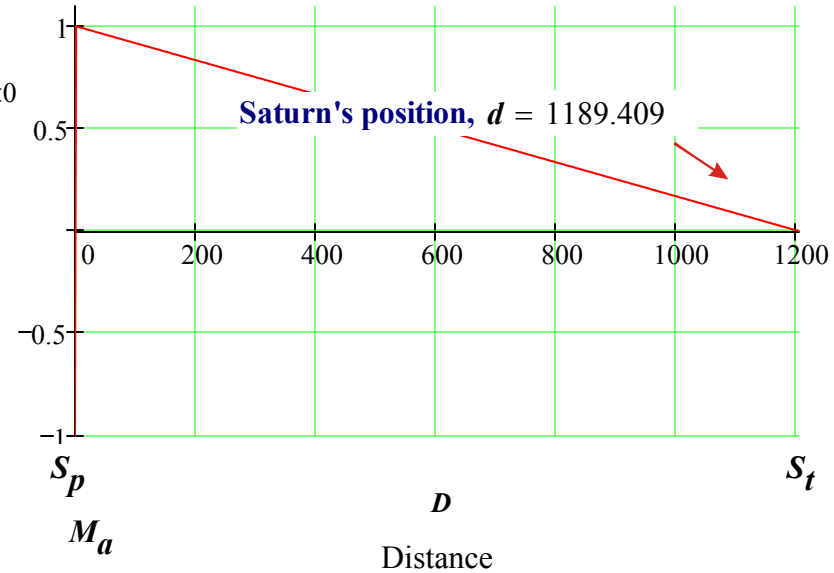
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 1.1894087 \times 10^3$ Saturn's corresponding distance to the Iapetus relative to S_p on the X axis of the graph

Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph

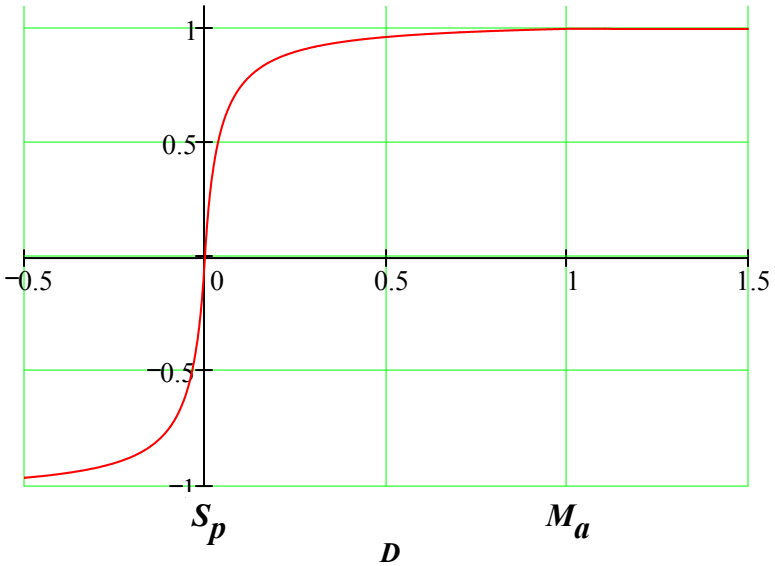


Distance

Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Left end of the Rotation Graph



Distance

$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - \left(\sqrt{0.29} - 0.5 \right)} - \left(\sqrt{0.29} - 0.5 \right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5 \right)} + \left(0.5 + \sqrt{0.29} \right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.01220728$ Saturn's corresponding Influenced Rotation by the Iapetus on the Y axis of the graph

$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 3.1393726 \times 10^{-8}$ Saturn's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_8 := if\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r \right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_8 = 3.418474 \times 10^{-7}$ **Saturn's end result Influenced Rotation by the Iapetus (p.d.)**
(Negative number means the reduction amount from Saturn's Free Rotation)

Part 9

Saturn's Total Rotation

$$\omega_S := \sum_{i=1}^{8} \omega i_i$$

$\omega_S = 2.19866562$ Saturn's Total Rotation (p.d.)

$$T := if\left(\omega i_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_S}, \frac{-1}{\omega_S}\right)\right)$$

$T = 0.4548$ **Saturn's Sidereal Rotation Period (day)**
If (T = 0 , Saturn's Synchronous Tropical Rotation)

Observation

$T_o := 0.4493$ **Saturn's Sidereal Rotation Period (day)**
If (*T* = 0 , Saturn's Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o)\cdot 200}{T + T_o}$$

$\%Diff = 1.2214$ Percentage deference between the calculation and the observation