

Uranus' Rotation Equations

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November 15, 2004 - Updated July 2012

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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts

	<u>Uranus</u>	<u>Sun</u>
Mass (kg)	$\underline{m} := 8.6832 \cdot 10^{25}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm ³)	$\rho := 1.27$	$\rho_s := 1.408$
Axis Tilt (deg)	$t := 97.77$	$t_s := 7.25$
Semi-major Axis (km)	$a := 2872460000$	
Orbit Eccentricity (deg)	$\underline{e} := 0.04716771$	
Orbit Inclination (degree), with respect to equator	$i := 6.48$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$$\omega_F = 0.14951229$$

Uranus' Free Rotation (per day)

Part 1

Uranus' Influenced Rotation by the influence of the Sun



$$q := a \cdot (1 - e)$$

$$q = 2736972639.7 \quad \text{Uranus' Perihelion Distance (km)}$$

$$Q := a \cdot (1 + e)$$

$$Q = 3007947360.3 \quad \text{Uranus' Aphelion Distance (km)}$$

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99680566 \quad \text{Uranus' Influenced Rotation Reduction Factor by Orbit Inclination}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m \cdot i_r \div M} \div \sqrt[6]{\rho}}{i_M}$$

$$\omega_{Mi} = 1.41865954 \quad \text{Uranus' Maximum Influenced Rotation by the Sun (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{St}}$$

$$S_t = 185291043211.9 \quad \text{Uranus' Start Influenced Rotation Distance to the Sun (km)}$$

$$M_a := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Ma}}$$

$$M_a = 329229740.2 \quad \text{Uranus' Maximum Influenced Rotation Distance to the Sun (km)}$$

$$S_p := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Sp}}$$

$$S_p = 175491441.1 \quad \text{Uranus' Stop Rotation Distance to the Sun (km)}$$

Calculating Uranus' average distance to the Sun, if ($q < S_p < Q$)

$$x := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{S_p - a}{e}, 0 \right), 0 \right)$$

$x = 0$ X value at Uranus' orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 2869262906.5$ Uranus' Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 2869262906.53$ Y value at the Uranus' orbit intersection with S_p Boundary (km)

$$\theta := \text{atan} \left(\frac{-x}{y} \right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Uranus' orbit out of S_p Boundary (rad)

$$s := a \cdot \int_0^\theta \sqrt{1 - e^2 \cdot \sin(\theta)^2} d\theta$$

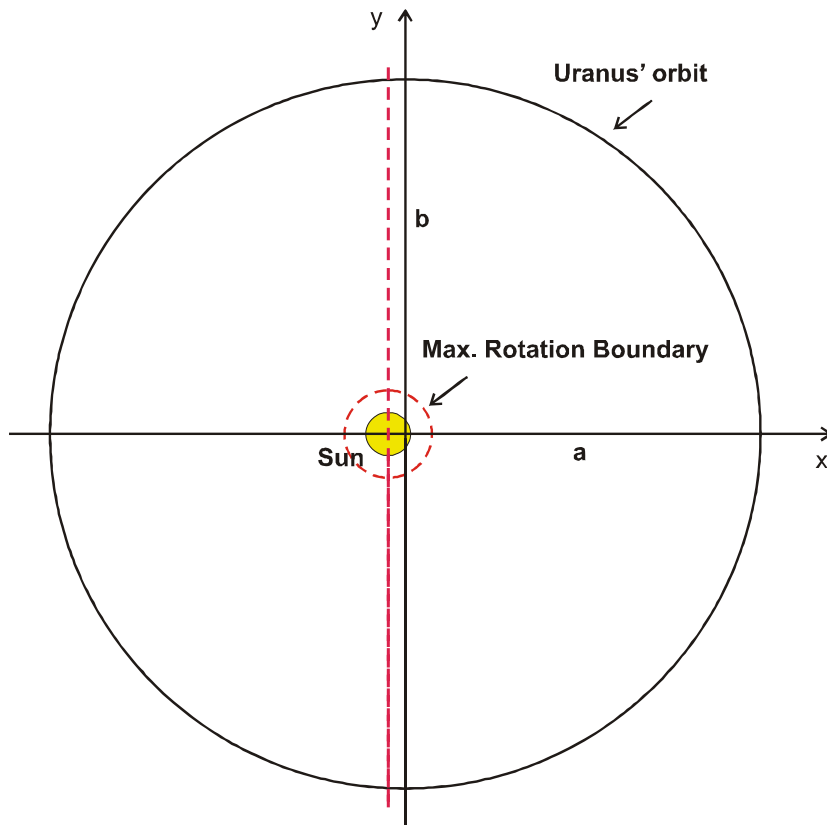
$s = 4509538975.07$ Half of Uranus' orbit out of S_p Boundary (km)

$$a_a := \text{if} \left[q < S_p, \text{if} \left[S_p < Q, a \frac{\int_{\pi - \frac{s}{a}}^\pi (1 - e \cdot \cos(E)) \cdot \sqrt{1 - e^2 \cdot \cos(E)^2} dE}{\int_{\pi - \frac{s}{a}}^\pi \sqrt{1 - e^2 \cdot \cos(E)^2} dE}, 0 \right], 0 \right]$$

$a_a = 0$ Uranus' average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a \cdot \int_0^\pi \sqrt{1 - e^2 \cdot \sin(\theta)^2} d\theta} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$n = 0$ Ratio of the Uranus' orbit out of S_p Boundary to the whole orbit

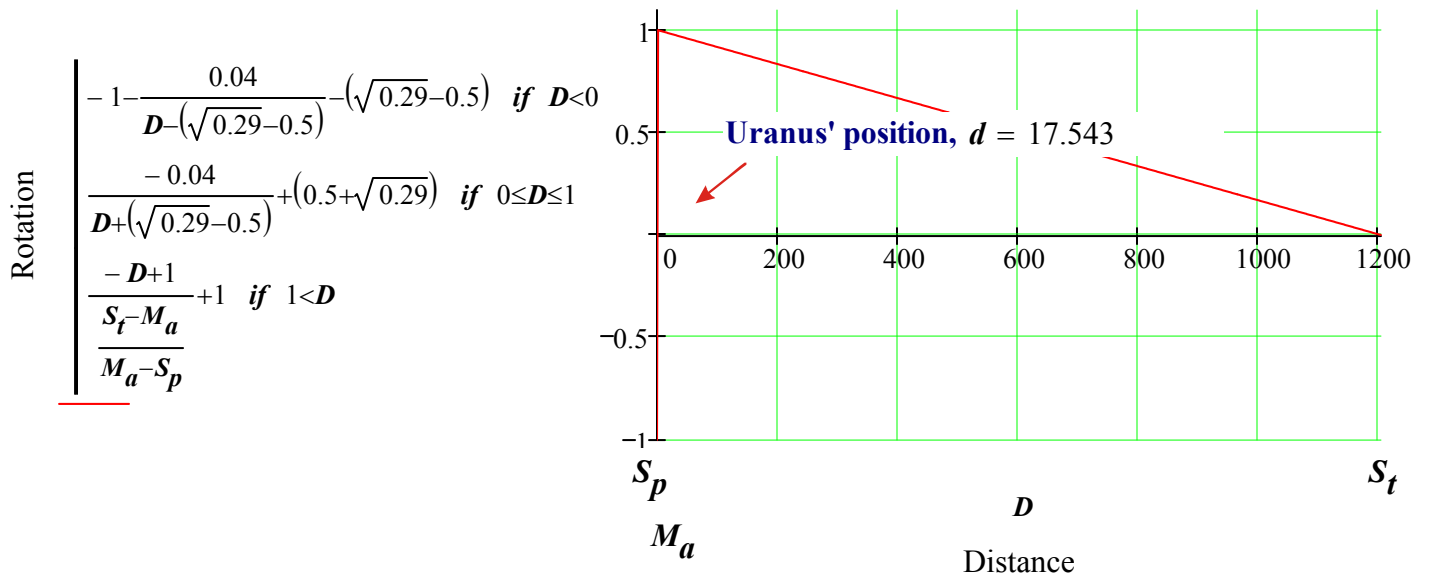


Uranus' orbit relative to the Max. Rotation Boundary

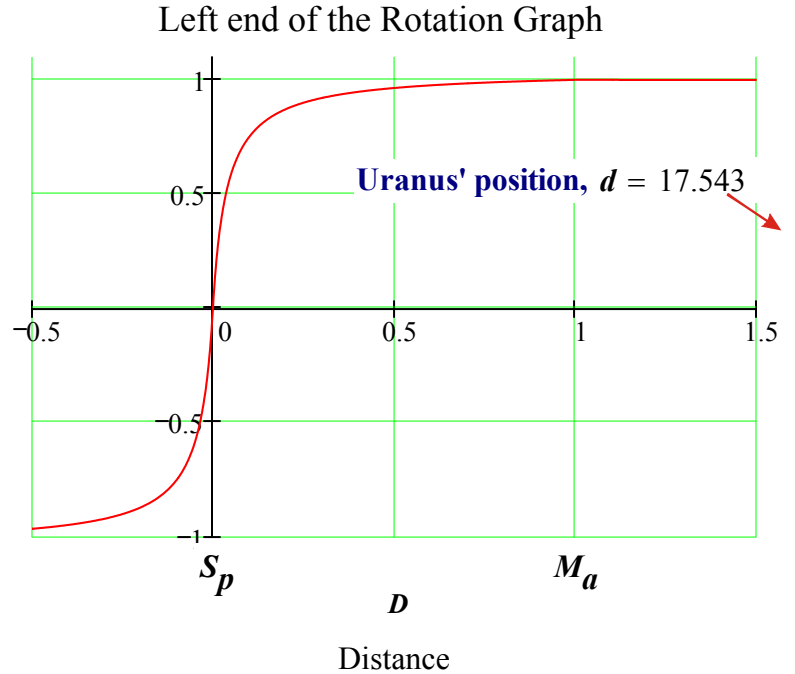
$$d := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p} \right), \frac{a - S_p}{M_a - S_p} \right)$$

$d = 17.54259397$ Uranus' corresponding distance to the Sun relative to S_p on the X axis of the graph

Non-proportional Rotation Graph



$$\text{Rotation} \left| \begin{array}{l} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 \text{ if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$



$$\omega(d) := \left| \begin{array}{l} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq d \leq 1 \\ \frac{-d+1}{S_t - M_a} + 1 \text{ if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$

$\omega(d) = 0.98624997$ Uranus' corresponding Influenced Rotation by the Sun on the Y axis of the graph

$$t_r := \text{if} \left(a < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t \cdot \omega_F}{90}, \frac{t \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 0.16242019$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega_1 := \text{if} \left[a > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot (\omega_{Mi} + \omega_F - t_r) \cdot \text{if} \left(q < S_p, \text{if} \left(Q > S_p, n, 0 \right), 1 \right) \right] \right]$$

$\omega_1 = 1.38624503$ Uranus' end result Rotation (p.d.)

Part 2

Uranus' Influenced Rotation by the influence of the [Miranda](#)

if ($q < S_t$)



Miranda's Facts

$a_m := 129900$	Miranda Semi-major Axis (km)
$e_m := 0.0013$	Miranda Orbit Eccentricity (degree)
$i_m := 4.338$	Miranda Orbit Inclination (degree)
$t_m := 0.162$	Miranda Axis Tilt (degree)
$m_m := 6.6 \cdot 10^{19}$	Miranda Mass (kg)

$$q := a_m \cdot (1 - e_m)$$

$q = 129731.1$ Miranda's Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$Q = 130068.9$ Miranda's Aphelion Distance (km)

$$i_r := \left(\left| \cos \left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.999998$ Miranda's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$\omega_{Mi} = 0.000004163$ Uranus' Maximum Influenced Rotation by the Miranda (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$S_t = 543671.2$ Uranus' Start Influenced Rotation Distance to the Miranda (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$M_a = 966$ Uranus' Maximum Influenced Rotation Distance to the Miranda (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

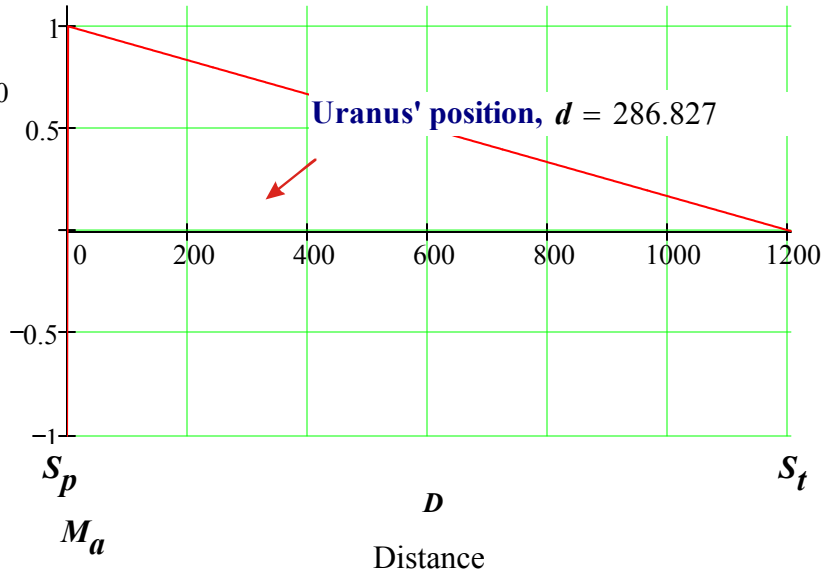
$S_p = 514.9$ Uranus' Stop Rotation Distance to the Miranda (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 286.82711943$ Uranus' corresponding distance to the Miranda relative to S_p on the X axis of the graph

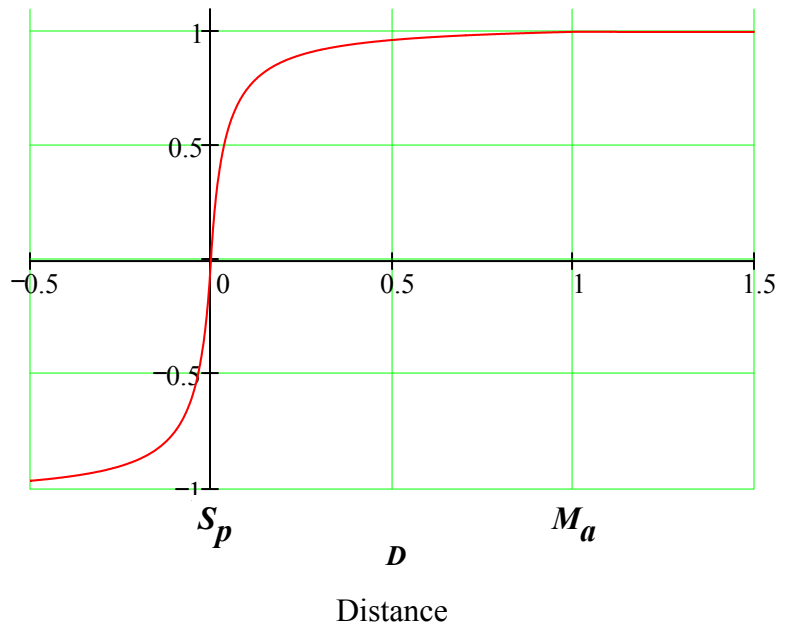
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.76242353$ Uranus' corresponding Influenced Rotation by the Miranda on the Y axis of the graph

$$t_r := \text{if} \left(a_m < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 1.52969 \times 10^{-7}$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

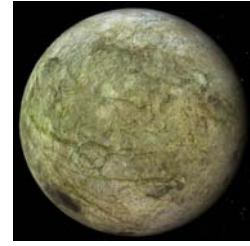
$$\omega i_2 := \text{if} \left[a_m < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_2 = 3.0206617 \times 10^{-6}$ **Uranus' end result Influenced Rotation by the Miranda (p.d.)**
(Negative number means the reduction amount from Uranus' Free Rotation)

Part 3

Uranus' Influenced Rotation by the influence of the [Ariel](#)

if ($q < S_t$)



Ariel's Facts

$$a_{mm} := 190900$$

Ariel Semi-major Axis (km)

$$e_{mm} := 0.0012$$

Ariel Orbit Eccentricity (degree)

$$i_{mm} := 0.041$$

Ariel Orbit Inclination (degree)

$$t_{mm} := 0.162$$

Ariel Axis Tilt (degree)

$$m_{mm} := 1.35 \cdot 10^{21}$$

Ariel Mass (kg)

$$q := a_m \cdot (1 - e_m)$$

$$q = 190670.9$$

Ariel's Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 191129.1$$

Ariel's Aphelion Distance (km)

$$i_{rr} := \left(\left| \cos \left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$$i_r = 0.999998$$

Ariel's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$$\omega_{Mi} = 0.000031133$$

Uranus' Maximum Influenced Rotation by the Ariel (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$$S_t = 4066270.2$$

Uranus' Start Influenced Rotation Distance to the Ariel (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$$M_a = 7225.1$$

Uranus' Maximum Influenced Rotation Distance to the Ariel (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

$$S_p = 3851.2$$

Uranus' Stop Rotation Distance to the Ariel (km)

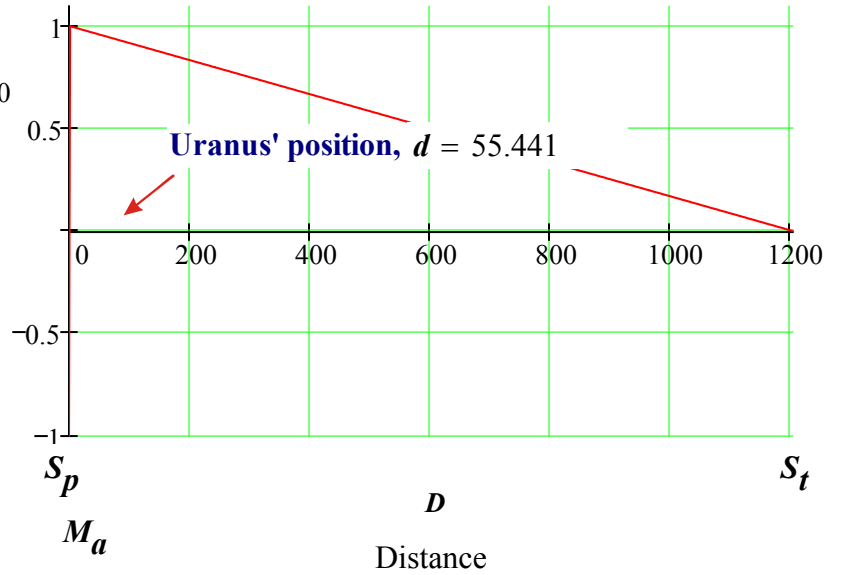
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$$d = 55.44099402$$

Uranus' corresponding distance to the Ariel relative to S_p on the X axis of the graph

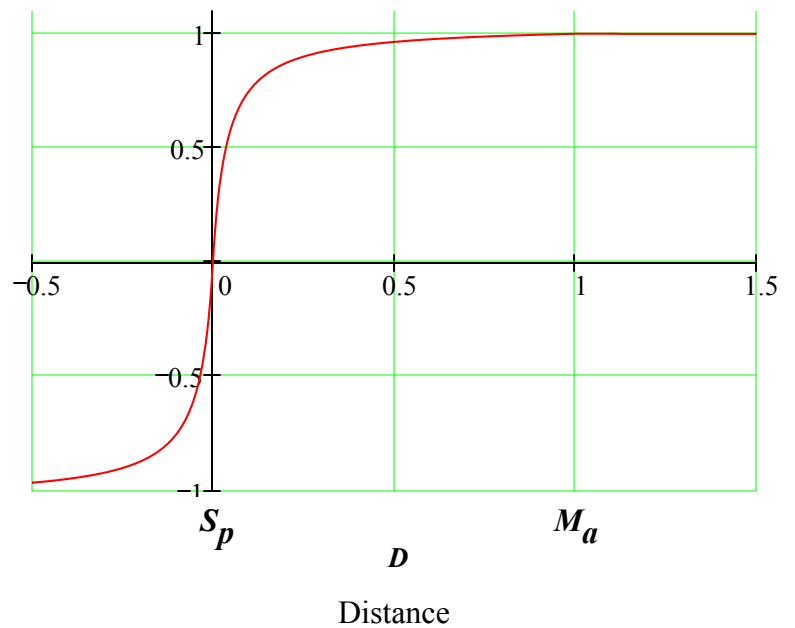
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.95474922$ Uranus' corresponding Influenced Rotation by the Ariel on the Y axis of the graph

$$t_r := \text{if} \left(a_m < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 1.3540997 \times 10^{-8}$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

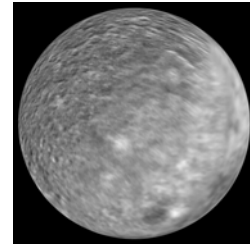
$$\omega i_3 := \text{if} \left[a_m < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_3 = 2.97106 \times 10^{-5}$ **Uranus' end result Influenced Rotation by the Ariel (p.d.)**
(Negative number means the reduction amount from Uranus' Free Rotation)

Part 4

Uranus' Influenced Rotation by the influence of the [Umbriel](#)

if ($q < S_t$)



Umbriel's Facts

$$a_m := 266000$$

Umbriel Semi-major Axis (km)

$$e_m := 0.0039$$

Umbriel Orbit Eccentricity (degree)

$$i_m := 0.128$$

Umbriel Orbit Inclination (degree)

$$t_m := 0.162$$

Umbriel Axis Tilt (degree)

$$m_m := 1.17 \cdot 10^{21}$$

Umbriel Mass (kg)

$$q := a_m \cdot (1 - e_m)$$

$$q = 264962.6$$

Umbriel's Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 267037.4$$

Umbriel's Aphelion Distance (km)

$$i_r := \left(\left| \cos \left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$$i_r = 0.999998$$

Umbriel's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$$\omega_{Mi} = 0.0000283$$

Uranus' Maximum Influenced Rotation by the Umbriel (p.d.)

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$$S_t = 3696275.2$$

Uranus' Start Influenced Rotation Distance to the Umbriel (km)

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$$M_a = 6567.6$$

Uranus' Maximum Influenced Rotation Distance to the Umbriel (km)

$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

$$S_p = 3500.8$$

Uranus' Stop Rotation Distance to the Umbriel (km)

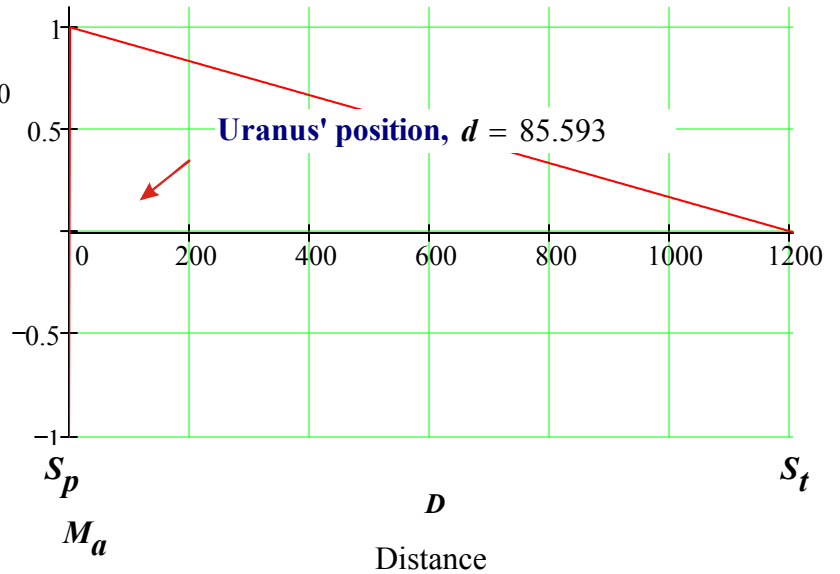
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$$d = 85.59256835$$

Uranus' corresponding distance to the Umbriel relative to S_p on the X axis of the graph

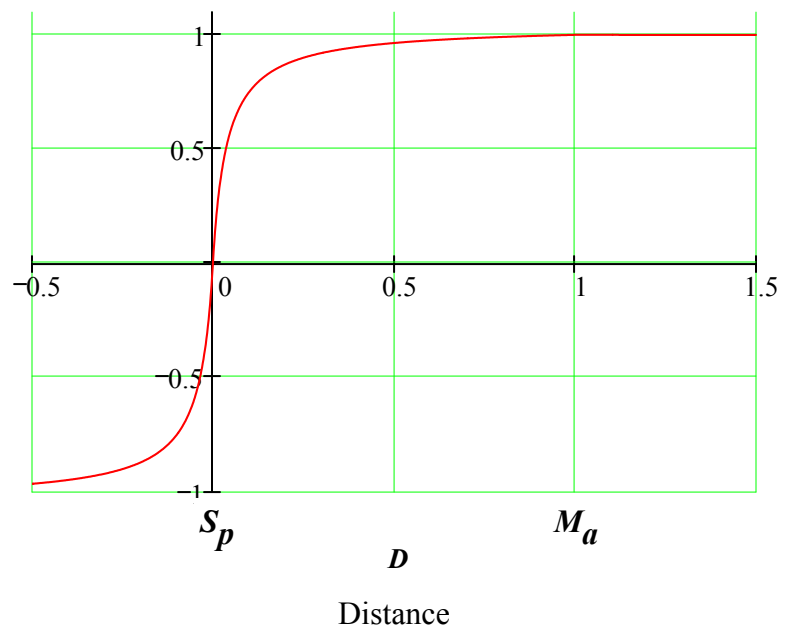
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.92968755$ Uranus' corresponding Influenced Rotation by the Umbriel on the Y axis of the graph

$$t_r := \text{if} \left(a_m < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 3.7419031 \times 10^{-8}$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_4 := \text{if} \left[a_m < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_4 = 2.6272837 \times 10^{-5}$ Uranus' end result Influenced Rotation by the Umbriel (p.d.)
(Negative number means the reduction amount from Uranus' Free Rotation)

Part 5

Uranus' Influenced Rotation by the influence of the [Titania](#)

if ($q < S_t$)



Titania's Facts

$a_m := 436300$ Titania Semi-major Axis (km)

$e_m := 0.0011$ Titania Orbit Eccentricity (degree)

$i_m := 0.079$ Titania Orbit Inclination (degree)

$t_m := 0.162$ Titania Axis Tilt (degree)

$m_m := 3.52 \cdot 10^{21}$ Titania Mass (kg)

$q := a_m \cdot (1 - e_m)$
 $q = 435820.1$ Titania's Perihelion Distance (km)

$Q := a_m \cdot (1 + e_m)$
 $Q = 436779.9$ Titania's Aphelion Distance (km)

$i_r := \left(\left| \cos \left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$
 $i_r = 0.999998$ Titania's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$
 $\omega_{Mi} = 0.000058978$ Uranus' Maximum Influenced Rotation by the Titania (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$
 $S_t = 7703158.5$ Uranus' Start Influenced Rotation Distance to the Titania (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$
 $M_a = 13687.2$ Uranus' Maximum Influenced Rotation Distance to the Titania (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$
 $S_p = 7295.8$ Uranus' Stop Rotation Distance to the Titania (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

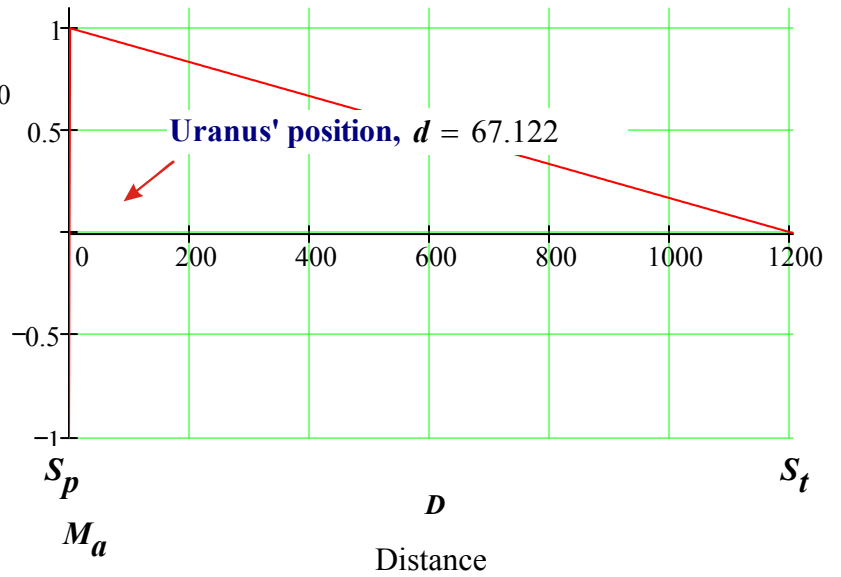
$$d = 67.12203386$$

Uranus' corresponding distance to the Titania relative to S_p on the X axis of the graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$

$$\frac{M_a - S_p}{M_a - S_p}$$

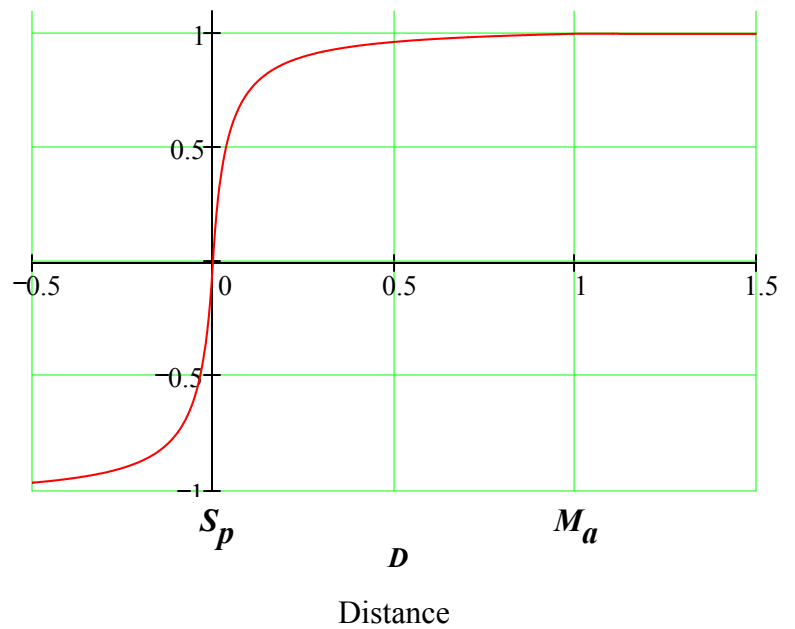
Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$

$$\frac{M_a - S_p}{M_a - S_p}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.94504007$ Uranus' corresponding Influenced Rotation by the Titania on the Y axis of the graph

$$t_r := \text{if} \left(a_m < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 4.8924612 \times 10^{-8}$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_5 := \text{if} \left[a_m < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_5 = 5.5687976 \times 10^{-5}$ **Uranus' end result Influenced Rotation by the Titania (p.d.)**
(Negative number means the reduction amount from Uranus' Free Rotation)

Part 6

Uranus' Influenced Rotation by the influence of the **Oberon**

if ($q < S_t$)



Oberon's Facts

a_{mm}	:= 583500	Oberon Semi-major Axis (km)
e_{mm}	:= 0.0014	Oberon Orbit Eccentricity (degree)
i_{mm}	:= 0.068	Oberon Orbit Inclination (degree)
t_{mm}	:= 0.16225	Oberon Axis Tilt (degree)
m_{mm}	:= $3.01 \cdot 10^{21}$	Oberon Mass (kg)

$$q := a_m \cdot (1 - e_m)$$

$$q = 582683.1 \quad \text{Oberon's Perihelion Distance (km)}$$

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 584316.9 \quad \text{Oberon's Aphelion Distance (km)}$$

$$i_r := \left(\left| \cos \left(\frac{t_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$$i_r = 0.999998 \quad \text{Oberon's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$$

$$\omega_{Mi} = 0.000053134 \quad \text{Uranus' Maximum Influenced Rotation by the Oberon (p.d.)}$$

$$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$$

$$S_t = 6939870.9 \quad \text{Uranus' Start Influenced Rotation Distance to the Oberon (km)}$$

$$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$$

$$M_a = 12330.9 \quad \text{Uranus' Maximum Influenced Rotation Distance to the Oberon (km)}$$

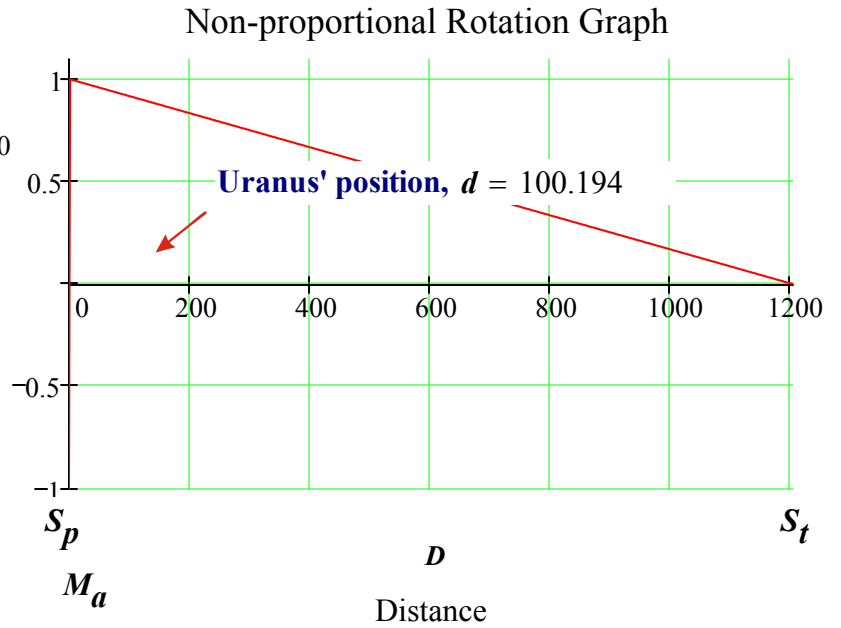
$$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$$

$$S_p = 6572.8 \quad \text{Uranus' Stop Rotation Distance to the Oberon (km)}$$

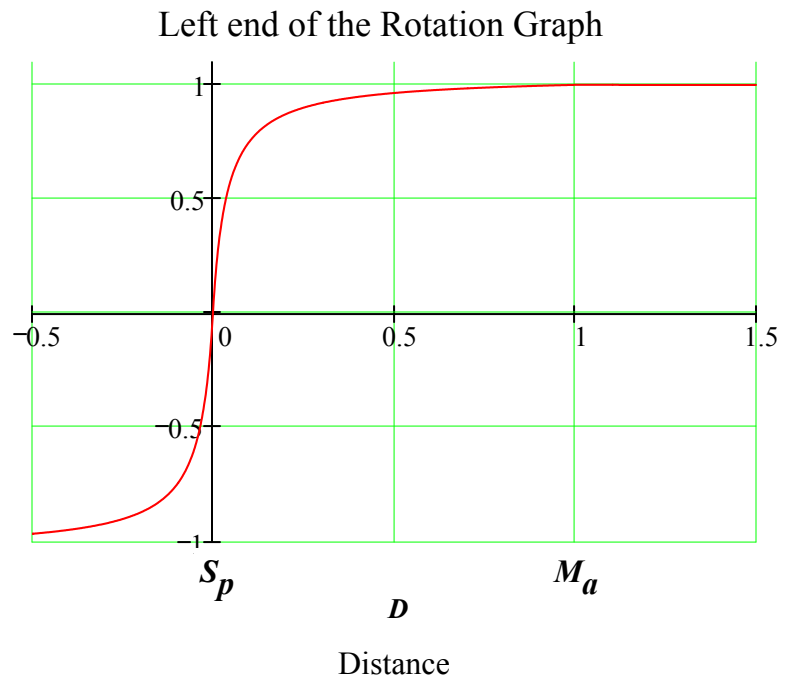
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 100.19405467$ Uranus' corresponding distance to the Oberon relative to S_p on the X axis of the graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p} \right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p} \right)}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.91755095$ Uranus' corresponding Influenced Rotation by the Oberon on the Y axis of the graph

$$t_r := \text{if} \left(a_m < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 3.6835938 \times 10^{-8}$ Uranus' Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_6 := \text{if} \left[a_m < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega i_6 = 4.8716612 \times 10^{-5}$ **Uranus' end result Influenced Rotation by the Oberon (p.d.)**
(Negative number means the reduction amount from Uranus' Free Rotation)

Part 7

Uranus' Total Rotation

$$\omega_s := \sum_{i=1}^6 \omega_i$$

$$\omega_s = 1.38640844 \quad \text{Uranus' Total Rotation (p.d.)}$$

$$T := \text{if} \left(\omega_1 \leq 0, 0, \text{if} \left(t \leq 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s} \right) \right)$$

$$T = -0.72129 \quad \text{Uranus' Sidereal Rotation Period (day)}$$

If (T = 0 , Uranus' Synchronous Tropical Rotation)

Observation

$$T_o := -0.71833 \quad \text{Uranus' Sidereal Rotation Period (day)}$$

If (T = 0 , Uranus' Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$$\%Diff = 0.4110 \quad \text{Percentage deference between the calculation and the observation}$$