Neptune's Rotation Equations

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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Neptune	Sun
Mass (kg)	$m := 1.0243 \cdot 10^{26}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho := 1.638$	$\rho_S := 1.408$
Axis Tilt (deg)	$t := 28.32$	$t_S := 7.25$
Semi-major Axis (km)	$a := 4495060000$	
Orbit Eccentricity (deg)	$e := 0.00858587$	
Orbit Inclination (degree), with respect to equator	$i := 6.43$	

$$\omega_F := f_R \div \sqrt[6]{m} \cdot \sqrt[2]{\rho}$$

$\omega_F = 0.16518633$ Neptune's Free Rotation (per day)

Part 1

Neptune's Influenced Rotation by the influence of the Sun

$$q := a \cdot (1 - e)$$

$q = 4456465999.2$ Neptune's Perihelion Distance (km)

$$Q := a \cdot (1 + e)$$

$Q = 4533654000.8$ Neptune's Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99685471$ Neptune's Influenced Rotation Reduction Factor by Orbit Inclination

$$\omega_{Mi} := \frac{\sqrt[6]{m \cdot i_r \div M} \div \sqrt[6]{\rho}}{i_M}$$

$\omega_{Mi} = 1.39772353$ Neptune's Maximum Influenced Rotation by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{St}}$$

$S_t = 190465307903.6$ Neptune's Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 338423502.6$ Neptune's Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m \cdot i_r \div M}}{i_{Sp}}$$

$S_p = 180392051.2$ Neptune's Stop Rotation Distance to the Sun (km)

Calculating Neptune's average distance to the Sun, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a}{e}, 0\right), 0\right)$$

$x = 0$ X value at Neptune's orbit intersection with S_p Boundary (km)

$$b := a\sqrt{1 - e^2}$$

$b = 4494894315.4$ Neptune's Semi-minor Axis (km)

$$y := b\sqrt{a^2 - x^2} \div a$$

$y = 4494894315.41$ Y value at the Neptune's orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Neptune's orbit out of S_p Boundary (rad)

$$s := a \cdot \int_0^{\theta} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta$$

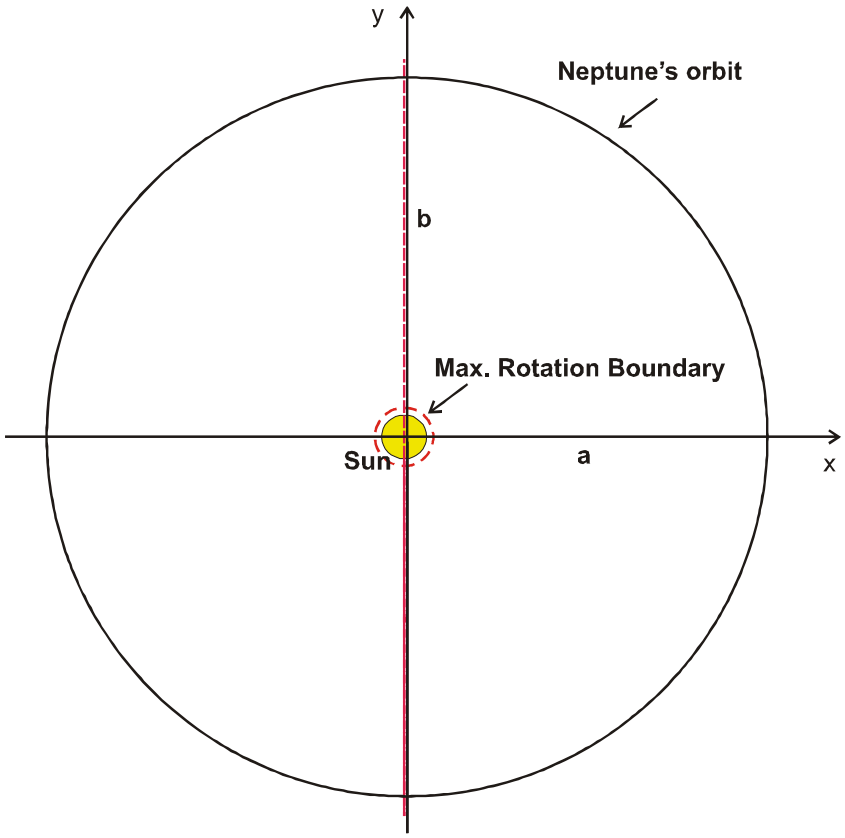
$s = 7060693608.95$ Half of Neptune's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a\frac{\int_{\pi - \frac{s}{a}}^{\pi} (1 - e \cdot cos(E)) \cdot \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}{\int_{\pi - \frac{s}{a}}^{\pi} \sqrt{1 - e^2 \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Neptune's average distance to the Sun outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{2 \cdot a \cdot \int_0^{\pi} \sqrt{1 - e^2 \cdot sin(\theta)^2}\, d\theta} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$n = 0$ Ratio of the Neptune's orbit out of S_p Boundary to the whole orbit

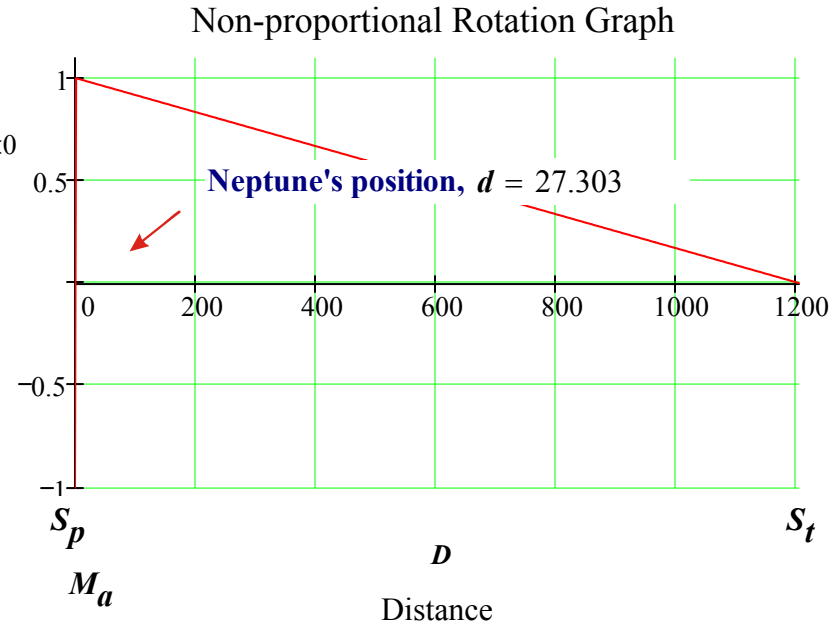


Neptune's orbit relative to the Max. Rotation Boundary

$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a - S_p}{M_a - S_p}\right), \frac{a - S_p}{M_a - S_p}\right)$$

$d = 27.30259017$ Neptune's corresponding distance to the Sun relative to S_p on the X axis of the graph

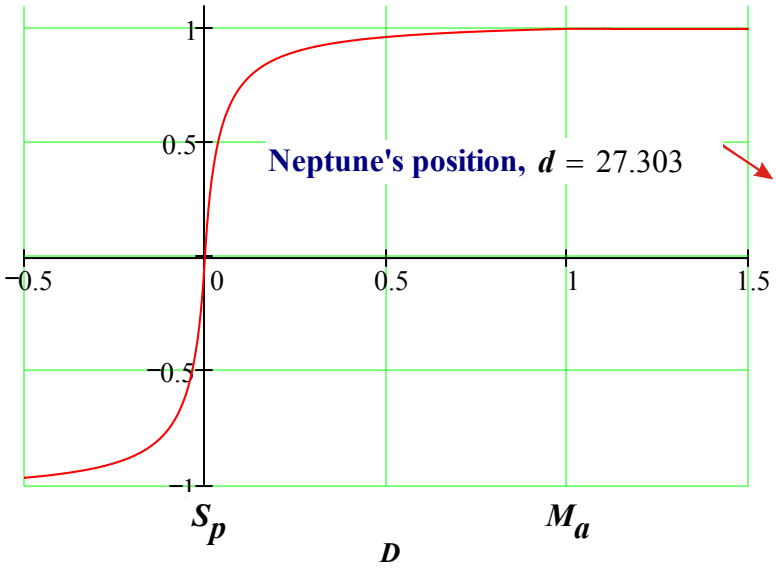
$$\text{Rotation} := \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1-\dfrac{0.04}{D-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } D<0 \\ \dfrac{-0.04}{D+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le D\le 1 \\ \dfrac{-D+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<D \end{cases}$$

Left end of the Rotation Graph



Distance

$$\omega(d) := \begin{cases} -1-\dfrac{0.04}{d-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } d<0 \\ \dfrac{-0.04}{d+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le d\le 1 \\ \dfrac{-d+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<d \end{cases}$$

$\omega(d) = 0.97813757$ Neptune's corresponding Influenced Rotation by the Sun on the Y axis of the graph

$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{t\cdot\omega_F}{90}, \frac{t\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 0.05197863$ Neptune's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_1 := if\left[a > M_a, \omega(d)\cdot\omega_{Mi}+\omega_F-t_r, \left[\omega(d)\cdot\left(\omega_{Mi}+\omega_F-t_r\right)\cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega i_1 = 1.48037359$ **Neptune's end result Rotation (p.d.)**

Part 2

Neptune's Influenced Rotation by the influence of the Despina

$if\left(q < S_t\right)$

Despina's Facts

$a_m := 52526$ Despina Semi-major Axis (km)

$e_m := 0.0001$ Despina Orbit Eccentricity (degree)

$i_m := 0.065$ Despina Orbit Inclination (degree)

$t_m := 0.008$ Despina Axis Tilt (degree)

$m_m := 2.2 \cdot 10^{18}$ Despina Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 52520.7$ Despina's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 52531.3$ Despina's Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 1$ Despina's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000000402$ Neptune's Maximum Influenced Rotation by the Despina (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 54781.1$ Neptune's Start Influenced Rotation Distance to the Despina (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 97.3$ Neptune's Maximum Influenced Rotation Distance to the Despina (km)

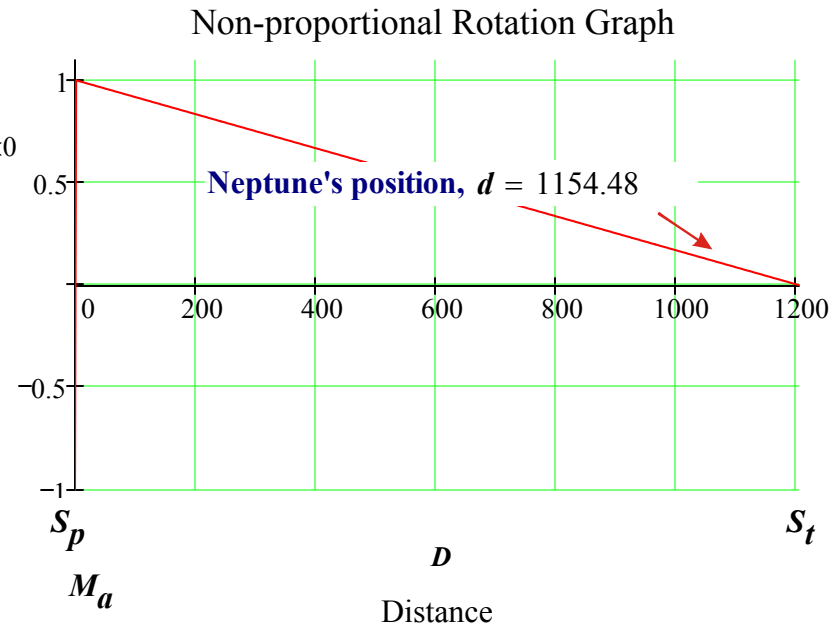
$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

$S_p = 51.9$ Neptune's Stop Rotation Distance to the Despina (km)

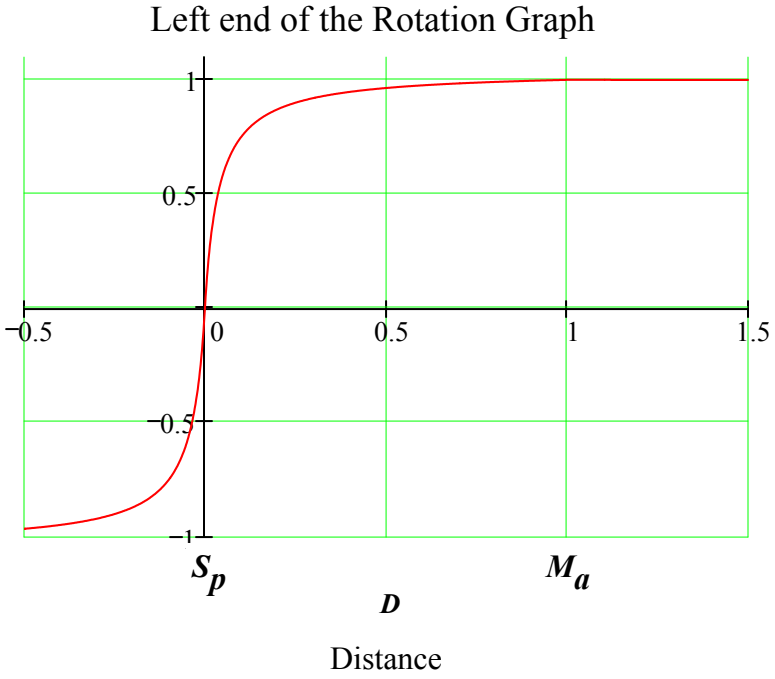
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 1.15447994 \times 10^3$ Neptune's corresponding distance to the Despina relative to S_p on the X axis of the graph

$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04\cdot\left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.0412397$ Neptune's corresponding Influenced Rotation by the Despina on the Y axis of the graph

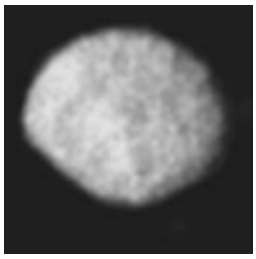
$$t_r := \text{if}\left(a_m < M_a, \text{if}\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), \text{if}\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 1.1973547\times10^{-11}$ Neptune's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_2 := \text{if}\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), \text{if}\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_2 = 1.6566784\times10^{-8}$ **Neptune's end result Influenced Rotation by the Despina (p.d.)**
(Negative number means the reduction amount from Neptune's Free Rotation)

Part 3



Neptune's Influenced Rotation by the influence of the Larissa

$if\left(q < S_t\right)$

Larissa's Facts

$a_m := 73548$ Larissa Semi-major Axis (km)

$e_m := 0.0014$ Larissa Orbit Eccentricity (degree)

$i_m := 0.201$ Larissa Orbit Inclination (degree)

$t_m := 0.048$ Larissa Axis Tilt (degree)

$m_m := 4.2 \cdot 10^{18}$ Larissa Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 73445$ Larissa's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 73651$ Larissa's Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.99999982$ Larissa's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000000619$ Neptune's Maximum Influenced Rotation by the Larissa (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 84304.1$ Neptune's Start Influenced Rotation Distance to the Larissa (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 149.8$ Neptune's Maximum Influenced Rotation Distance to the Larissa (km)

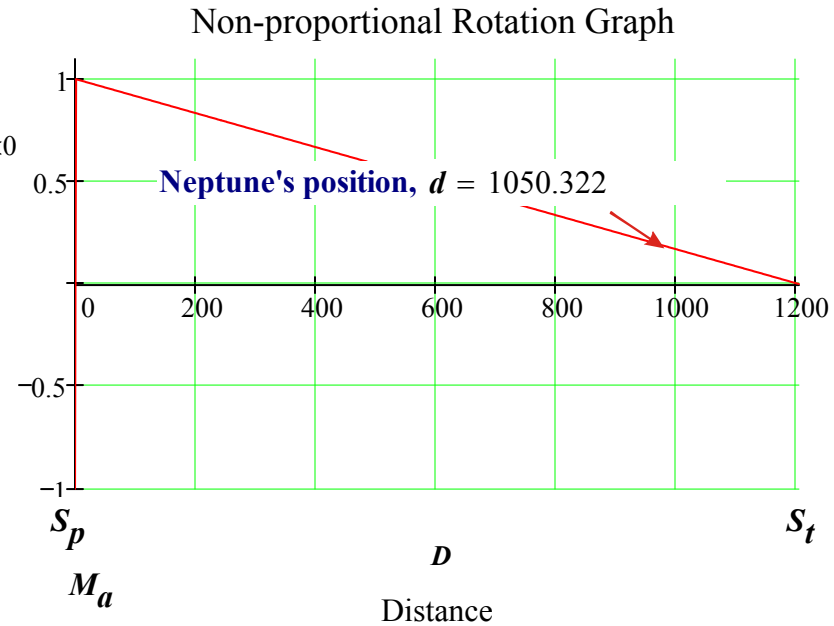
$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

$S_p = 79.8$ Neptune's Stop Rotation Distance to the Larissa (km)

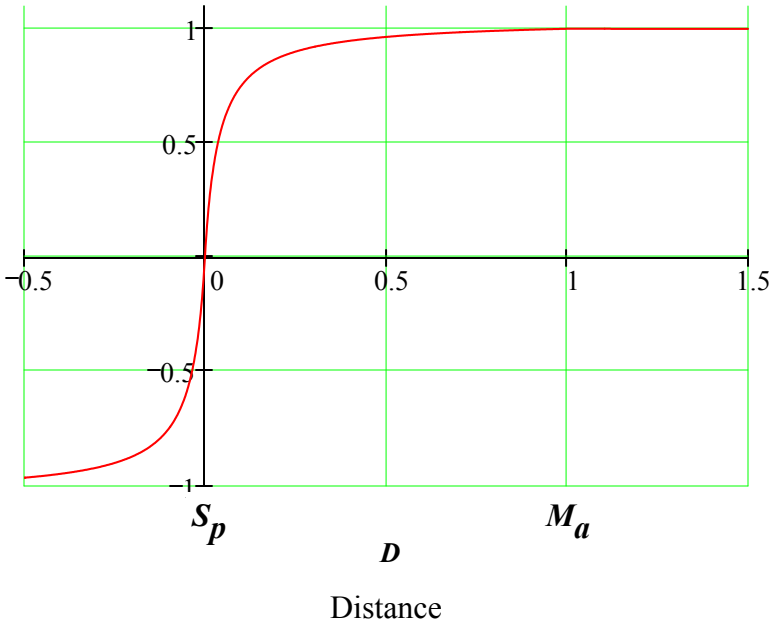
$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 1.05032248 \times 10^3$ Neptune's corresponding distance to the Larissa relative to S_p on the X axis of the graph

$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1\cdot\left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04\cdot\left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.12781427$ Neptune's corresponding Influenced Rotation by the Larissa on the Y axis of the graph

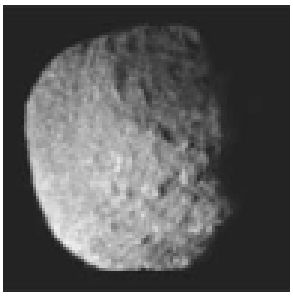
$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega_{Mi}}{90}\right), if\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{i_m\cdot\omega_F}{90}, \frac{i_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)$$

$t_r = 1.7659854\times10^{-10}$ Neptune's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_3 := if\left[a_m < M_a, \omega(d)\cdot\left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d)\cdot\omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_3 = 7.8897376\times10^{-8}$ **Neptune's end result Influenced Rotation by the Larissa (p.d.)**
(Negative number means the reduction amount from Neptune's Free Rotation)

Part 4



Neptune's Influenced Rotation by the influence of the Proteus

$if\left(q < S_t\right)$

Proteus' Facts

$a_m := 117647$ Proteus Semi-major Axis (km)

$e_m := 0.0004$ Proteus Orbit Eccentricity (degree)

$i_m := 0.039$ Proteus Orbit Inclination (degree)

$t_m := 0.548$ Proteus Axis Tilt (degree)

$m_m := 5.03 \cdot 10^{19}$ Proteus Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 117599.9$ Proteus' Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 117694.1$ Proteus' Aphelion Distance (km)

$i_r := \left(\left|\cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.99997713$ Proteus' Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000003238$ Neptune's Maximum Influenced Rotation by the Proteus (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 441292.3$ Neptune's Start Influenced Rotation Distance to the Proteus (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 784.1$ Neptune's Maximum Influenced Rotation Distance to the Proteus (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

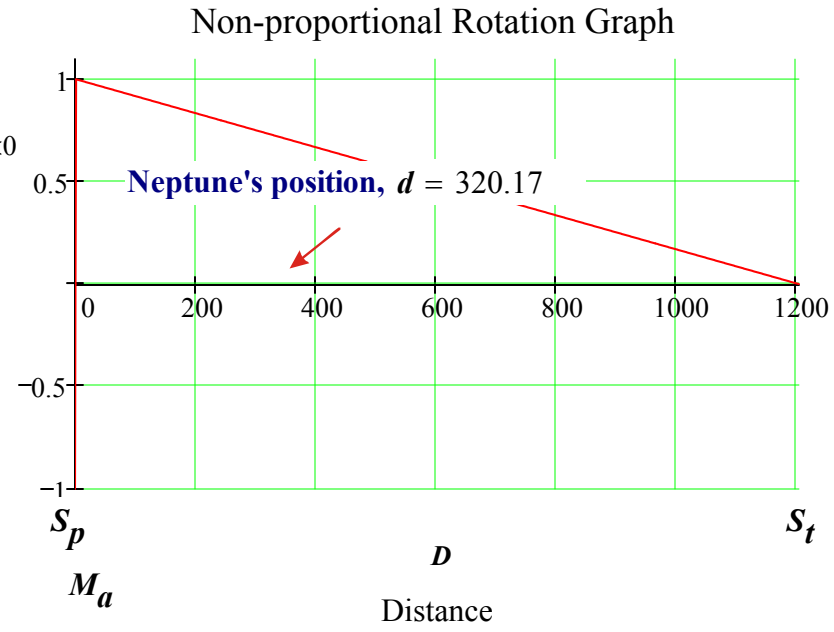
$S_p = 418$ Neptune's Stop Rotation Distance to the Proteus (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 320.1704728$ Neptune's corresponding distance to the Proteus relative to S_p on the X axis of the graph

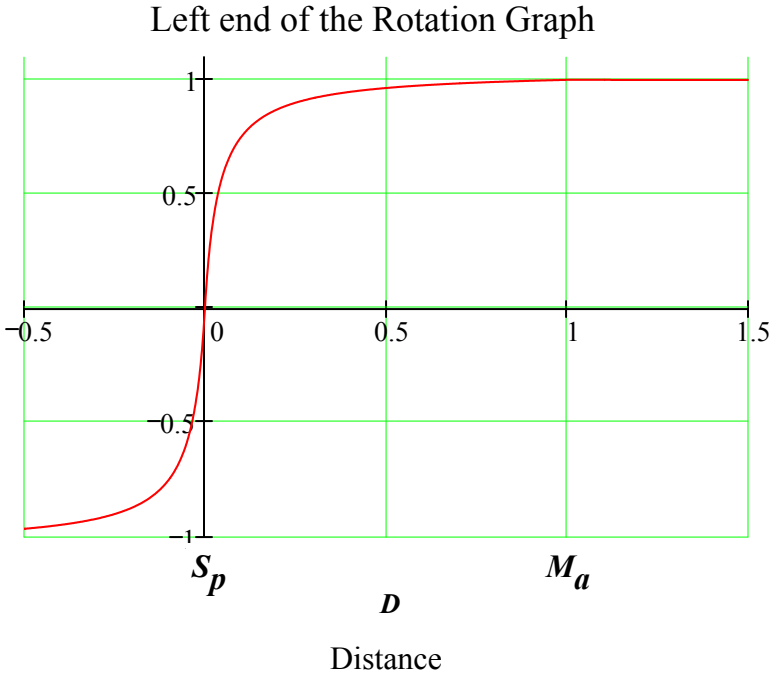
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.73470889$ Neptune's corresponding Influenced Rotation by the Proteus on the Y axis of the graph

$$t_r := \text{if}\left(a_m < M_a, \text{if}\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), \text{if}\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 1.0310248 \times 10^{-9}$ Neptune's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_4 := \text{if}\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), \text{if}\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_4 = 2.3782569 \times 10^{-6}$ **Neptune's end result Influenced Rotation by the Proteus (p.d.)**
(Negative number means the reduction amount from Neptune's Free Rotation)

Part 5

Neptune's Influenced Rotation by the influence of the Triton

$if\left(q < S_t\right)$

Triton's Facts

$a_m := 354760$ Triton Semi-major Axis (km)

$e_m := 0.000016$ Triton Orbit Eccentricity (degree)

$i_m := 156.834$ Triton Orbit Inclination (degree)

$t_m := 0.511$ Triton Axis Tilt (degree)

$m_m := 2.14 \cdot 10^{22}$ Triton Mass (kg)

$q := a_m \cdot \left(1 - e_m\right)$

$q = 354754.3$ Triton's Perihelion Distance (km)

$Q := a_m \cdot \left(1 + e_m\right)$

$Q = 354765.7$ Triton's Aphelion Distance (km)

$i_r := \left(\left|cos\left(\frac{t_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$

$i_r = 0.99998011$ Triton's Orbit Inclination Reduction Factor

$\omega_{Mi} := \sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho} \div i_M \div \sqrt{M \div m_m}$

$\omega_{Mi} = 0.000183187$ Neptune's Maximum Influenced Rotation by the Triton (p.d.)

$S_t := \sqrt[6]{m_m \cdot i_r \div m} \div i_{St} \div \sqrt{M \div m_m}$

$S_t = 24962632.6$ Neptune's Start Influenced Rotation Distance to the Triton (km)

$M_a := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma} \div \sqrt{M \div m_m}$

$M_a = 44354.2$ Neptune's Maximum Influenced Rotation Distance to the Triton (km)

$S_p := \sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp} \div \sqrt{M \div m_m}$

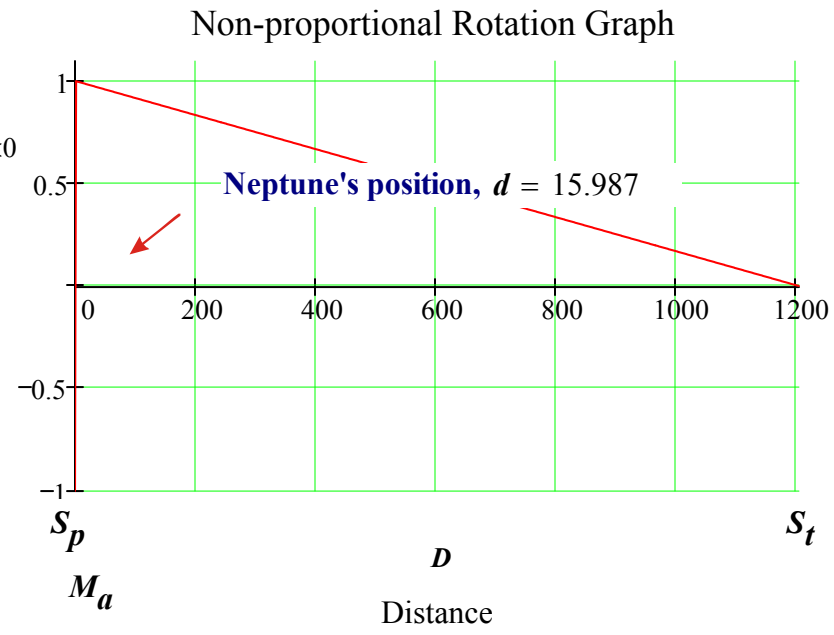
$S_p = 23642.4$ Neptune's Stop Rotation Distance to the Triton (km)

$$d := \frac{a_m - S_p}{M_a - S_p}$$

$d = 15.98689825$ Neptune's corresponding distance to the Triton relative to S_p on the X axis of the graph

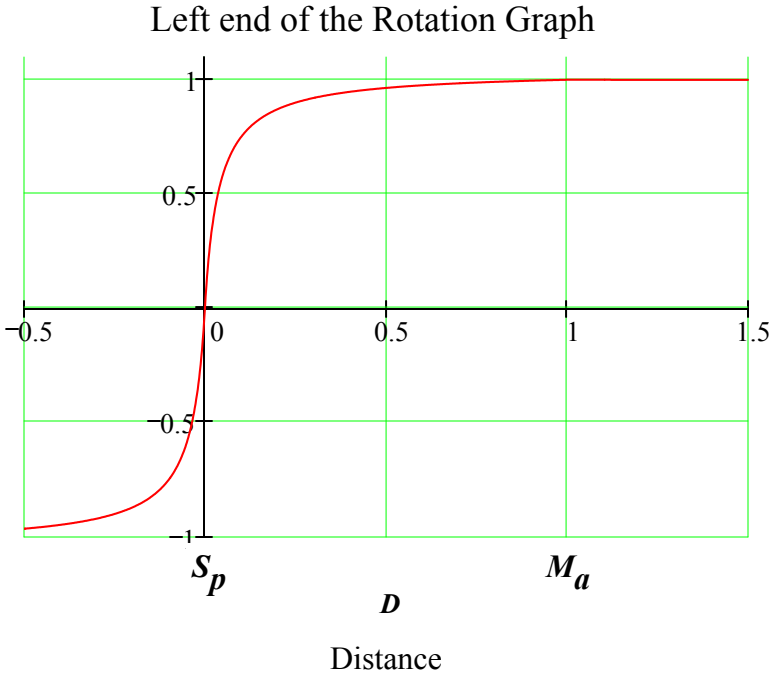
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 \cdot \left(\frac{S_p}{M_a - S_p}\right) - \frac{0.04 \cdot \left(\frac{S_p}{M_a - S_p}\right)}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \frac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \frac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.98754305$ Neptune's corresponding Influenced Rotation by the Triton on the Y axis of the graph

$$t_r := if\left(a_m < M_a, if\left(\omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{i_m \cdot \omega_F}{90}, \frac{i_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 3.1524596 \times 10^{-4}$ Neptune's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega i_5 := if\left[a_m < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega i_5 = -1.343404 \times 10^{-4}$ **Neptune's end result Influenced Rotation by the Triton (p.d.)**
(Negative number means the reduction amount from Neptune's Free Rotation)

Part 6

Neptune's Total Rotation

$$\omega_S := \sum_{i=1}^{5} \omega i_i$$

$\omega_S = 1.48024172$ Neptune's Total Rotation (p.d.)

$$T := if\left(\omega i_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_S}, \frac{-1}{\omega_S}\right)\right)$$

$T = 0.67557$ **Neptune's Sidereal Rotation Period (day)**
If (T = 0 , Neptune's Synchronous Tropical Rotation)

Observation

$T_O := 0.67125$ **Neptune's Sidereal Rotation Period (day)**
If (*T* = 0 , Neptune's Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_O)\cdot 200}{T + T_O}$$

$\%Diff = 0.6408$ Percentage deference between the calculation and the observation