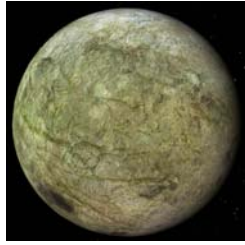


Ariel's Rotation Equations

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Constants

$$f_R := 2791.826$$

Free Rotation Constant

$$i_M := 0.127036$$

Maximum Influenced Rotation Constant (for planets and moons only)

$$i_{St} := 1.0121647 \cdot 10^{-12}$$

Start Influenced Rotation Distance Constant

$$i_{Ma} := 5.6964797 \cdot 10^{-10}$$

Maximum Influenced Rotation Distance Constant

$$i_{Sp} := 1.0686849 \cdot 10^{-9}$$

Stop Rotation Distance Constant

Facts

Ariel

Saturn

Sun

Mass (kg)

$$m_m := 1.35 \cdot 10^{21}$$

$$m_s := 8.6832 \cdot 10^{25}$$

$$M := 1.9891 \cdot 10^{30}$$

Density (g/cm³)

$$\rho_m := 1.665$$

$$\rho_s := 1.408$$

Axis Tilt (deg)

$$t_m := 0.010$$

$$t := 97.77$$

$$t_s := 7.25$$

Semi-major Axis (km)

$$a_m := 190900$$

$$a := 2872460000$$

Orbit Eccentricity (deg)

$$e_m := 0.0012$$

$$e_s := 0.04716771$$

Orbit Inclination (deg),
with respect to equator

$$i_m := 0.041$$

$$i := 6.48$$

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$$\omega_F = 1.08361042$$

Ariel's Free Rotational Speed (per day)

Part 1

Ariel's Influenced Rotation by the influence of the [Saturn](#)



$$q := a_m \cdot (1 - e_m)$$

$$q = 190670.9 \quad \text{Ariel's Perihelion Distance (km)}$$

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 191129.1 \quad \text{Ariel's Aphelion Distance (km)}$$

$$i_r := \left(\left| \cos\left(\frac{i_m \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99999987 \quad \text{Ariel's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m \div i_M}}{\sqrt[3]{M \div m}}$$

$$\omega_{Mi} = 0.04022112 \quad \text{Ariel's Maximum Influenced Rotational Speed by the Saturn (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$$S_t = 5495816118.8 \quad \text{Ariel's Start Influenced Rotation Distance to the Saturn (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$$M_a = 9765103 \quad \text{Ariel's Maximum Influenced Rotation Distance to the Saturn (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$$S_p = 5205155.5 \quad \text{Ariel's Stop Influenced Rotation Distance to the Saturn (km)}$$

Calculating Ariel's average distance to the Saturn, if ($q < S_p < Q$)

$$x := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{S_p - a_m}{e_m}, 0 \right), 0 \right)$$

$x = 0$ X value at Ariel's orbit intersection with S_p Boundary (km)

$$b := a_m \sqrt{1 - e_m^2}$$

$b = 190899.9$ Ariel's Semi-minor Axis (km)

$$y := b \sqrt{a_m^2 - x^2} \div a_m$$

$y = 190899.86$ Y value at the Ariel's orbit intersection with S_p Boundary (km)

$$\theta := \text{atan} \left(\frac{-x}{y} \right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Ariel's orbit out of S_p Boundary (rad)

$$P_o := 2 \cdot a_m \cdot \int_0^\pi \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$P_o = 1199459.64$ Ariel's Orbital Perimeter (km)

$$s_m := a_m \cdot \int_0^\theta \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$s = 299864.9$ Half of the Ariel's orbit out of S_p Boundary (km)

$$a_a := \text{if} \left[q < S_p, \text{if} \left[S_p < Q, a_m \frac{\int_{\pi - (s \div a_m)}^\pi \frac{\sqrt{1 - e_m^2 \cdot \cos(E)^2}}{1 \div (1 - e_m \cdot \cos(E))} dE}{\int_{\pi - (s \div a_m)}^\pi \sqrt{1 - e_m^2 \cdot \cos(E)^2} dE}, 0 \right], 0 \right]$$

$a_a = 0$ Ariel's average distance outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$$n = 0$$

Temporal ratio of the Ariel's orbit out of S_p Boundary to the whole orbit

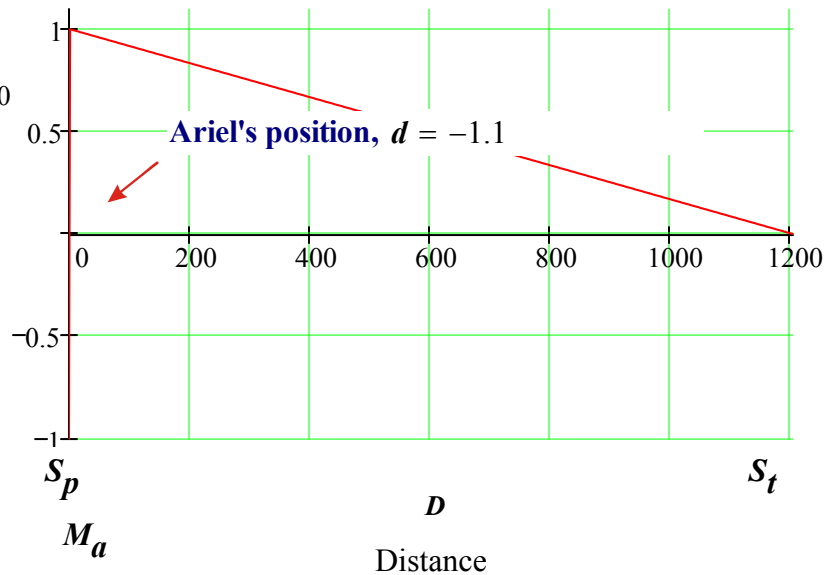
$$d := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p} \right), \frac{a_m - S_p}{M_a - S_p} \right)$$

$$d = -1.0996301$$

Ariel's corresponding distance on x axis of the graph

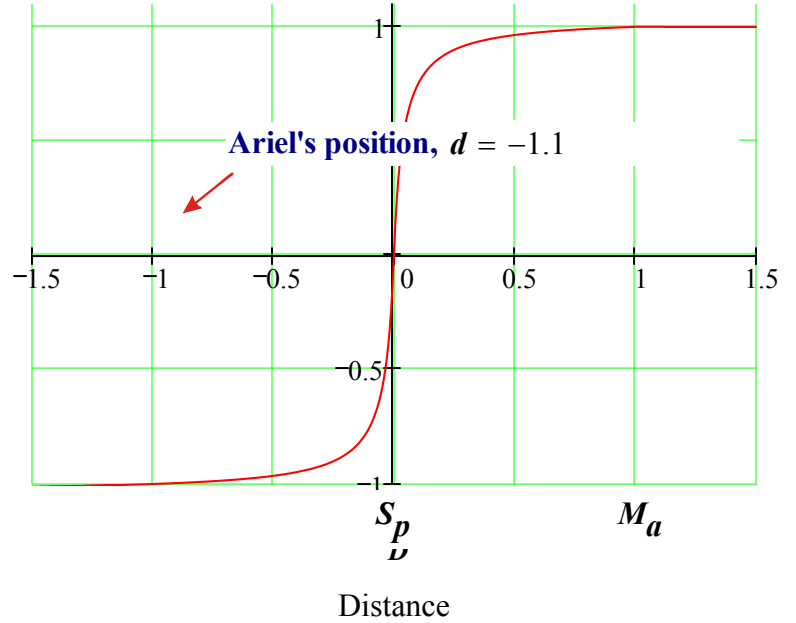
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph



Left end of the Rotation Graph

$$\text{Rotation} \left| \begin{array}{l} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 \text{ if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$



$$\omega(d) := \left| \begin{array}{l} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq d \leq 1 \\ \frac{-d+1}{S_t - M_a} + 1 \text{ if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$

$\omega(d) = -1.00337162$ Ariel's corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := \text{if} \left[a_m < M_a, \left(\text{if} \left(\omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90} \right) \right), \left(\text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right) \right]$$

$t_r = 0.00000447$ Ariel's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega_{c1} := \text{if} \left[a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot (\omega_{Mi} + \omega_F - t_r) \cdot \text{if} (q < S_p, \text{if} (Q > S_p, n, 0), 1) \right] \right]$$

$\omega_{c1} = 0$ Ariel's end result Influenced Rotation by the Saturn (p.d.)

Part 2

Ariel's Influenced Rotation by the influence of the Sun



$$q_v := a \cdot (1 - e)$$

$$q = 2736972639.7 \quad \text{Ariel/Saturn's Perihelion Distance to the Sun (km)}$$

$$Q_v := a \cdot (1 + e)$$

$$Q = 3007947360.3 \quad \text{Ariel/Saturn's Aphelion Distance to the Sun (km)}$$

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99680566 \quad \text{Ariel/Saturn's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$$\omega_{Mi} = 0.21423253 \quad \text{Ariel's Maximum Influenced Rotational Speed by the Sun (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$$S_t = 29272743042.4 \quad \text{Ariel's Start Influenced Rotation Distance to the Sun (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$$M_a = 52012538.9 \quad \text{Ariel's Maximum Influenced Rotation Distance to the Sun (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

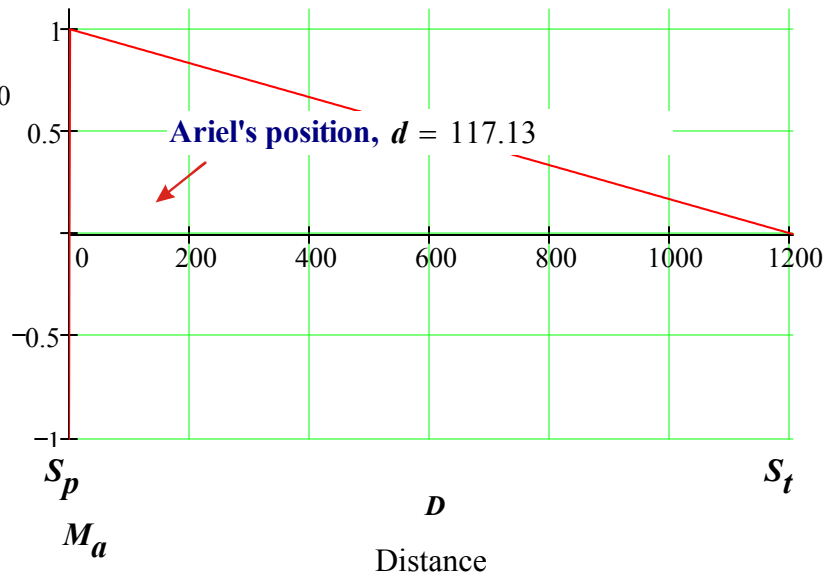
$$S_p = 27724577.4 \quad \text{Ariel's Stop Influenced Rotation Distance to the Sun (km)}$$

$$d := \frac{a - S_p}{M_a - S_p}$$

$d = 117.12532653$ Ariel's corresponding distance to the Sun relative to S_p on the X axis of the graph

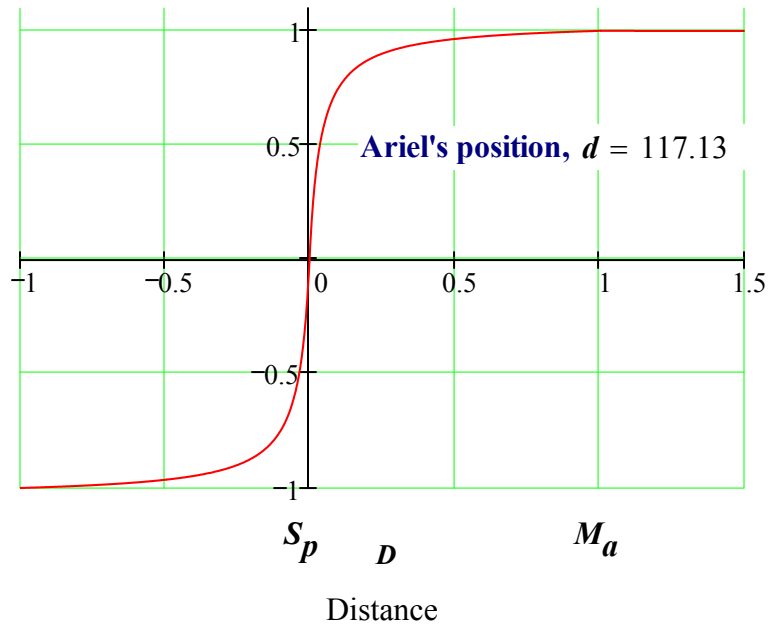
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 & \text{if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

$\omega(d) = 0.90347786$ Ariel's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 97.78$ Ariel's Axis Tilt with respect to the Sun (deg)

$$t_r := \text{if} \left(a < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 0.21028605$ Ariel's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega c_2 := \text{if} \left[a < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega c_2 = -0.0167317$ **Ariel's end result Influenced Rotation by the Sun (p.d.)**
(Negative number means the reduction amount from Ariel's Free Rotation)

Part 3

Ariel's Total Rotation

$$\omega_s := \sum_{i=1}^2 \omega_i$$

$$\omega_s = -0.0167317 \quad \text{Ariel's Total Rotation (p.d.)}$$

$$T := \text{if} \left(\omega_1 \leq 0, 0, \text{if} \left(t \leq 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s} \right) \right)$$

$$T = 0.0000 \quad \text{Ariel's Sidereal Rotation Period (day)}$$

If (T = 0 , Ariel's Synchronous Tropical Rotation)

Observation

$$T_o := 0.0000 \quad \text{Ariel's Sidereal Rotation Period (day)}$$

If (T = 0 , Ariel's Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$$\%Diff = 0.0000 \quad \text{Percentage difference between the calculation and the observation}$$