Calypso's Rotation Equations

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Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

Facts	Calypso	Saturn	Sun
Mass (kg)	$m_m := 4.0 \cdot 10^{15}$	$m := 5.6846 \cdot 10^{26}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm^3)	$\rho_m := 1.000$		$\rho_s := 1.408$
Axis Tilt (deg)	$t_m := 0.016$	$t := 26.73$	$t_s := 7.25$
Semi-major Axis (km)	$a_m := 294700$	$a := 1433530000$	
Orbit Eccentricity (deg)	$e_m := 0.0005$	$e := 0.0541506$	
Orbit Inclination (deg), with respect to equator	$i_m := 1.473$	$i := 5.51$	

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$\omega_F = 7.0072081$ Calypso's Free Rotational Speed (per day)

Part 1

Calypso's Influenced Rotation by the influence of the Saturn

$$q := a_m \cdot (1 - e_m)$$

$q = 294552.7$ Calypso's Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$Q = 294847.3$ Calypso's Aphelion Distance (km)

$$i_r := \left(\left| \cos\left(\frac{i_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99983477$ Calypso's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m} \div i_M}{\sqrt[3]{M \div m}}$$

$\omega_{Mi} = 0.00717743$ Calypso's Maximum Influenced Rotational Speed by the Saturn (p.d.)

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$S_t = 900833712.3$ Calypso's Start Influenced Rotation Distance to the Saturn (km)

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$M_a = 1600623.8$ Calypso's Maximum Influenced Rotation Distance to the Saturn (km)

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$S_p = 853190.8$ Calypso's Stop Influenced Rotation Distance to the Saturn (km)

Calculating Calypso's average distance to the Saturn, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a_m}{e_m}, 0\right), 0\right)$$

$x = 0$ X value at Calypso's orbit intersection with S_p Boundary (km)

$$b := a_m\sqrt{1 - e_m^{\,2}}$$

$b = 294700$ Calypso's Semi-minor Axis (km)

$$y := b\sqrt{a_m^{\,2} - x^2} \div a_m$$

$y = 294699.96$ Y value at the Calypso's orbit intersection with S_p Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Calypso's orbit out of S_p Boundary (rad)

$$P_o := 2\cdot a_m\cdot\int_0^{\pi}\sqrt{1 - e_m^{\,2}\cdot sin(\theta)^2}\,d\theta$$

$P_o = 1851654.59$ Calypso's Orbital Perimeter (km)

$$s := a_m\cdot\int_0^{\theta}\sqrt{1 - e_m^{\,2}\cdot sin(\theta)^2}\,d\theta$$

$s = 462913.6$ Half of the Calypso's orbit out of S_p Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a_m\frac{\displaystyle\int_{\pi-(s\div a_m)}^{\pi}\frac{\sqrt{1 - e_m^{\,2}\cdot cos(E)^2}}{1 \div \left(1 - e_m\cdot cos(E)\right)}\,dE}{\displaystyle\int_{\pi-(s\div a_m)}^{\pi}\sqrt{1 - e_m^{\,2}\cdot cos(E)^2}\,dE}, 0\right], 0\right]$$

$a_a = 0$ Calypso's average distance outside S_p Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^{\,3}}{a^3}}$$

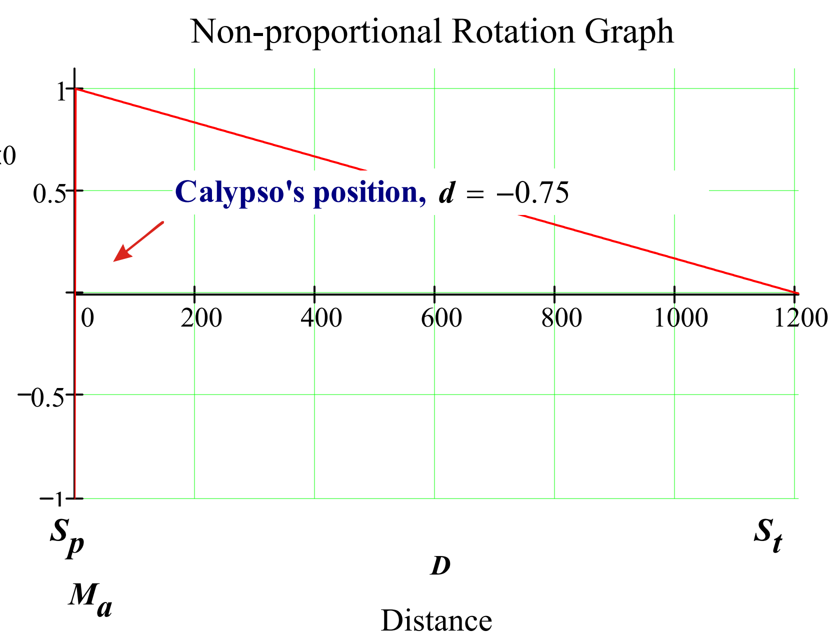
$n = 0$ Temporal ratio of the Calypso's orbit out of S_p Boundary to the whole orbit

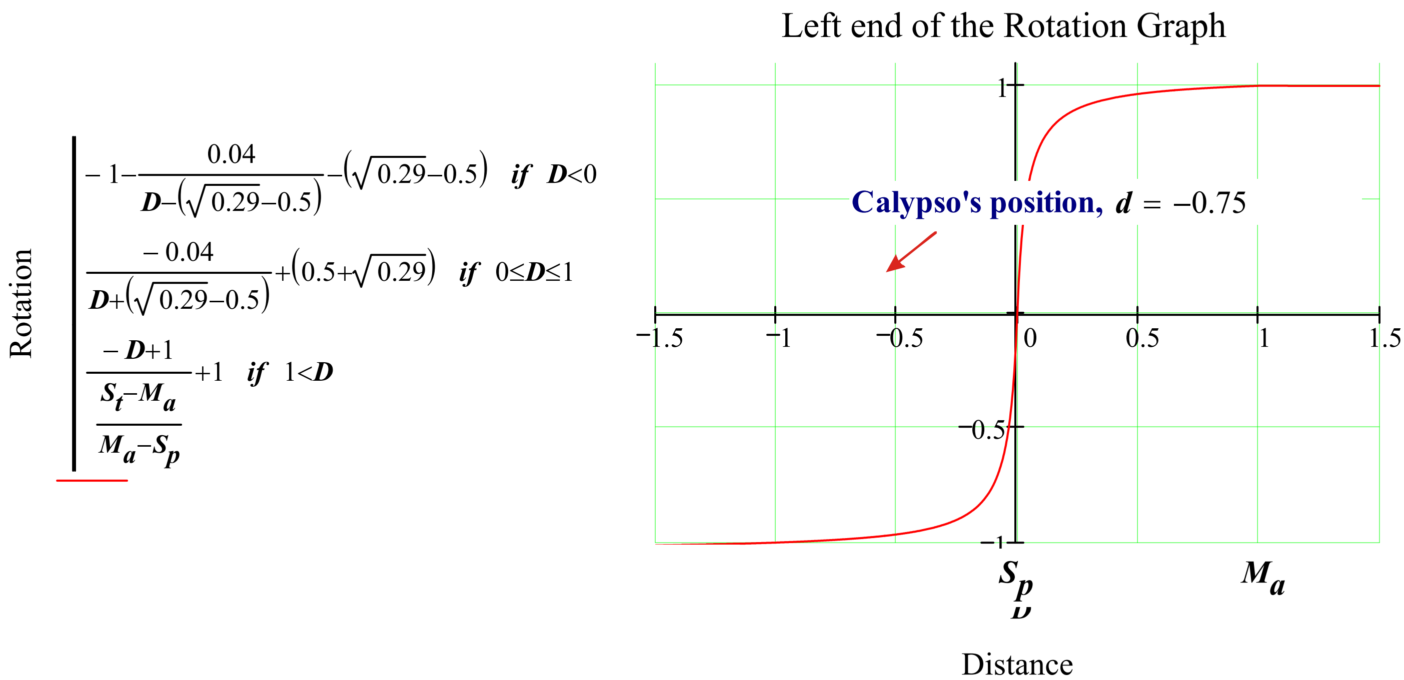
$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p}\right), \frac{a_m - S_p}{M_a - S_p}\right)$$

$d = -0.74721179$ Calypso's corresponding distance on x axis of the graph

Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$





$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = -0.9876083$ Calypso's corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := if\left[a_m < M_a, \left(if\left(\omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90}\right)\right), \left(if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)\right]$$

$t_r = 0.00000128$ Calypso's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega c_1 := if\left[a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot \left(\omega_{Mi} + \omega_F - t_r\right) \cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega c_1 = 0$ **Calypso's end result Influenced Rotation by the Saturn (p.d.)**

Part 2

Calypso's Influenced Rotation by the influence of the Sun

$q := a \cdot (1 - e)$

$q = 1355903490.4$ Calypso/Saturn's Perihelion Distance to the Sun (km)

$Q := a \cdot (1 + e)$

$Q = 1511156509.6$ Calypso/Saturn's Aphelion Distance to the Sun (km)

$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$

$i_r = 0.99768973$ Calypso/Saturn's Orbit Inclination Reduction Factor

$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$

$\omega_{Mi} = 0.02795587$ Calypso's Maximum Influenced Rotational Speed by the Sun (p.d.)

$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$

$S_t = 3508719298.3$ Calypso's Start Influenced Rotation Distance to the Sun (km)

$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$

$M_a = 6234379.8$ Calypso's Maximum Influenced Rotation Distance to the Sun (km)

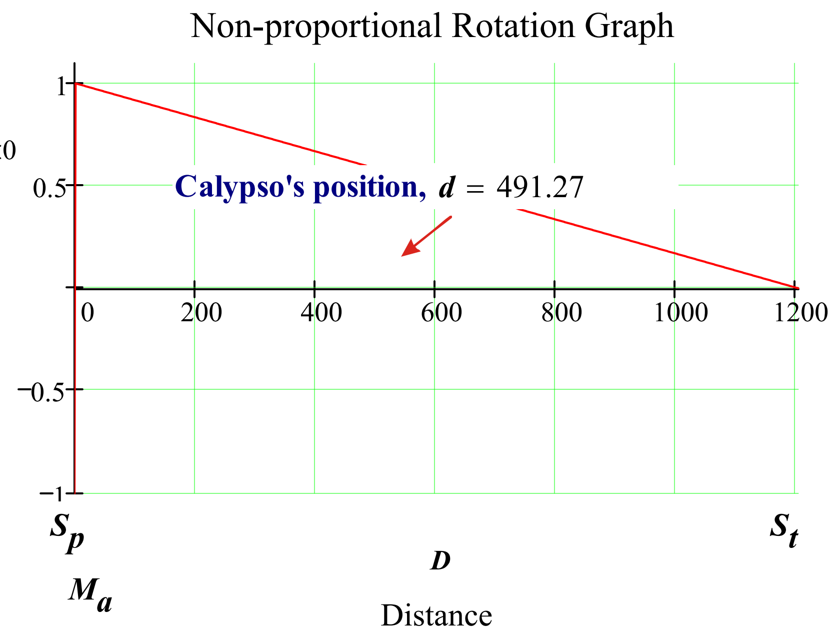
$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$

$S_p = 3323151.5$ Calypso's Stop Influenced Rotation Distance to the Sun (km)

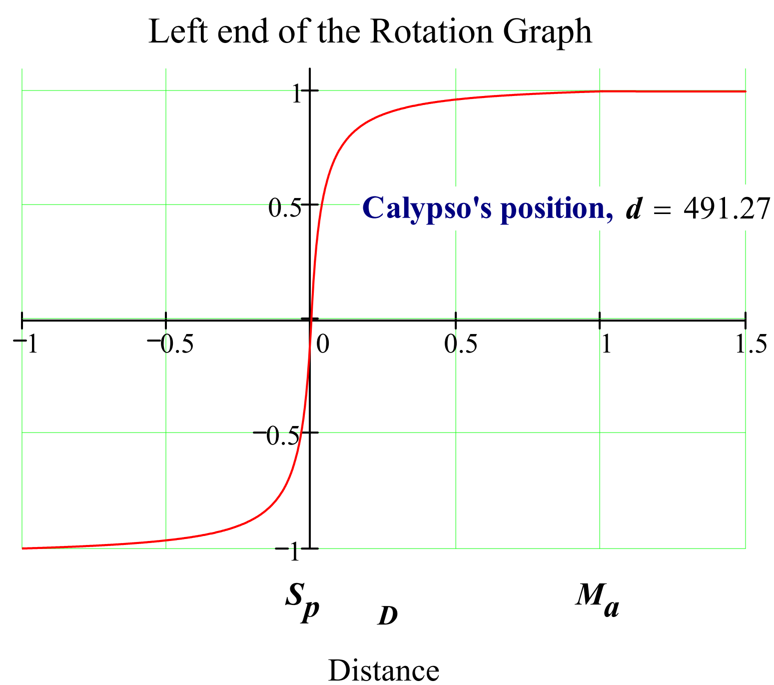
$$d := \frac{a - S_p}{M_a - S_p}$$

$d = 491.27264483$ Calypso's corresponding distance to the Sun relative to S_p on the X axis of the graph

$$\text{Rotation} \quad \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} \quad \begin{cases} -1 - \dfrac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \dfrac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.59249057$ Calypso's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 26.746$ Calypso's Axis Tilt with respect to the Sun (deg)

$$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 0.00492233$ Calypso's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega c_2 := if\left[a < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega c_2 = 0.01164126$ **Calypso's end result Influenced Rotation by the Sun (p.d.)**
(Negative number means the reduction amount from Calypso's Free Rotation)

Part 3

Calypso's Total Rotation

$$\omega_s := \sum_{i=1}^{2} \omega c_i$$

$\omega_s = 0.01164126$ Calypso's Total Rotation (p.d.)

$$T := if\left(\omega c_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s}\right)\right)$$

$T = 0.0000$ **Calypso's Sidereal Rotation Period (day)**
If (T = 0 , Calypso's Synchronous Tropical Rotation)

Observation

$T_o := 0.0000$ **Calypso's Sidereal Rotation Period (day)**
If (*T* = 0 , Calypso's Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o)\cdot 200}{T + T_o}$$

$\%Diff = 0.0000$ Percentage deference between the calculation and the observation