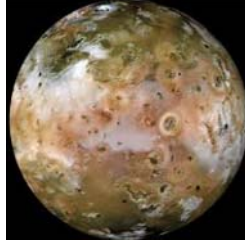


Io's Rotation Equations

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Constants

$$f_R := 2791.826$$

Free Rotation Constant

$$i_M := 0.127036$$

Maximum Influenced Rotation Constant (for planets and moons only)

$$i_{St} := 1.0121647 \cdot 10^{-12}$$

Start Influenced Rotation Distance Constant

$$i_{Ma} := 5.6964797 \cdot 10^{-10}$$

Maximum Influenced Rotation Distance Constant

$$i_{Sp} := 1.0686849 \cdot 10^{-9}$$

Stop Rotation Distance Constant

Facts

Io

Jupiter

Sun

Mass (kg)

$$m_m := 8.93 \cdot 10^{22}$$

$$m_j := 1.8986 \cdot 10^{27}$$

$$M := 1.9891 \cdot 10^{30}$$

Density (g/cm³)

$$\rho_m := 3.528$$

$$\rho_s := 1.408$$

Axis Tilt (deg)

$$t_m := 0.00$$

$$t := 3.13$$

$$t_s := 7.25$$

Semi-major Axis (km)

$$a_m := 421800$$

$$a := 778570000$$

Orbit Eccentricity (deg)

$$e_m := 0.0041$$

$$e_j := 0.09341233$$

Orbit Inclination (deg),
with respect to equator

$$i_m := 0.036$$

$$i := 6.09$$

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$$\omega_F = 0.78435149$$

Io's Free Rotational Speed (per day)

Part 1

Io's Influenced Rotation by the influence of the **Jupiter**



$$q := a_m \cdot (1 - e_m)$$

$$q = 420070.6 \quad \text{Io's Perihelion Distance (km)}$$

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 423529.4 \quad \text{Io's Aphelion Distance (km)}$$

$$i_r := \left(\left| \cos\left(\frac{i_m \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.9999999 \quad \text{Io's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m \div i_M}}{\sqrt[3]{M \div m}}$$

$$\omega_{Mi} = 0.1193476 \quad \text{Io's Maximum Influenced Rotational Speed by the Jupiter (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$$S_t = 18481792474.8 \quad \text{Io's Start Influenced Rotation Distance to the Jupiter (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$$M_a = 32838909.2 \quad \text{Io's Maximum Influenced Rotation Distance to the Jupiter (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$$S_p = 17504334.5 \quad \text{Io's Stop Influenced Rotation Distance to the Jupiter (km)}$$

Calculating Io's average distance to the Jupiter, if ($q < S_p < Q$)

$$x := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{S_p - a_m}{e_m}, 0 \right), 0 \right)$$

$$x = 0 \quad \text{X value at Io's orbit intersection with } S_p \text{ Boundary (km)}$$

$$b := a_m \sqrt{1 - e_m^2}$$

$$b = 421796.5 \quad \text{Io's Semi-minor Axis (km)}$$

$$y := b \sqrt{a_m^2 - x^2} \div a_m$$

$$y = 421796.45 \quad \text{Y value at the Io's orbit intersection with } S_p \text{ Boundary (km)}$$

$$\theta := \text{atan} \left(\frac{-x}{y} \right) + \frac{\pi}{2}$$

$$\theta = 1.57079633 \quad \text{Half-angle of the Io's orbit out of } S_p \text{ Boundary (rad)}$$

$$P_o := 2 \cdot a_m \cdot \int_0^\pi \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$$P_o = 2650236.42 \quad \text{Io's Orbital Perimeter (km)}$$

$$s_m := a_m \cdot \int_0^\theta \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$$s = 662559.1 \quad \text{Half of the Io's orbit out of } S_p \text{ Boundary (km)}$$

$$a_a := \text{if} \left[q < S_p, \text{if} \left[S_p < Q, a_m \frac{\int_{\pi - (s \div a_m)}^\pi \frac{\sqrt{1 - e_m^2 \cdot \cos(E)^2}}{1 \div (1 - e_m \cdot \cos(E))} dE}{\int_{\pi - (s \div a_m)}^\pi \sqrt{1 - e_m^2 \cdot \cos(E)^2} dE} \right], 0 \right], 0$$

$$a_a = 0 \quad \text{Io's average distance outside } S_p \text{ Boundary (km)}$$

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$$n = 0$$

Temporal ratio of the Io's orbit out of S_p Boundary to the whole orbit

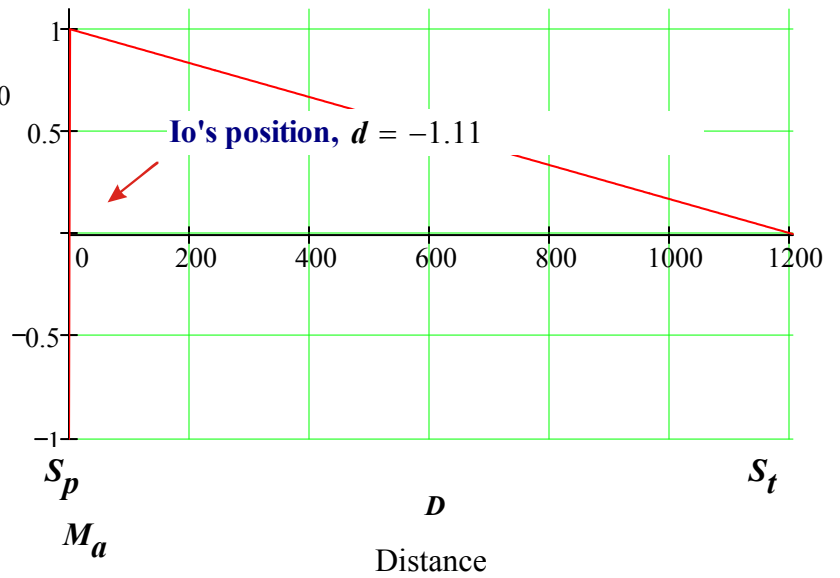
$$d := \text{if} \left(q < S_p, \text{if} \left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p} \right), \frac{a_m - S_p}{M_a - S_p} \right)$$

$$d = -1.11398815$$

Io's corresponding distance on x axis of the graph

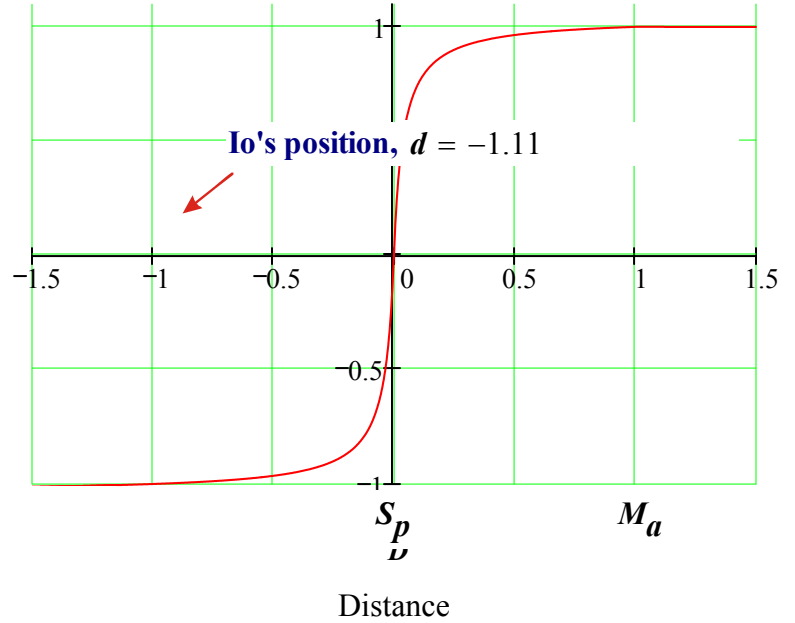
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph



Left end of the Rotation Graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D+1}{S_t - M_a} + 1 & \text{if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d+1}{S_t - M_a} + 1 & \text{if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

$\omega(d) = -1.00380946$ Io's corresponding Influenced Rotation by the Jupiter on the Y axis of the graph

$$t_r := \text{if} \left[a_m < M_a, \left(\text{if} \left(\omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90} \right) \right), \left(\text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right) \right]$$

$t_r = 0$ Io's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega_{c1} := \text{if} \left[a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot (\omega_{Mi} + \omega_F - t_r) \cdot \text{if} (q < S_p, \text{if} (Q > S_p, n, 0), 1) \right] \right]$$

$\omega_{c1} = 0$ Io's end result Influenced Rotation by the Jupiter (p.d.)

Part 2

Io's Influenced Rotation by the influence of the [Sun](#)



$$q_v := a \cdot (1 - e)$$

$$q = 705841962.2 \quad \text{Io/Jupiter's Perihelion Distance to the Sun (km)}$$

$$Q_v := a \cdot (1 + e)$$

$$Q = 851298037.8 \quad \text{Io/Jupiter's Aphelion Distance to the Sun (km)}$$

$$i_r := \left(\left| \cos\left(\frac{i \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99717824 \quad \text{Io/Jupiter's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$$\omega_{Mi} = 0.38017162 \quad \text{Io's Maximum Influenced Rotational Speed by the Sun (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$$S_t = 58872175606.7 \quad \text{Io's Start Influenced Rotation Distance to the Sun (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$$M_a = 104605547.8 \quad \text{Io's Maximum Influenced Rotation Distance to the Sun (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

$$S_p = 55758566.4 \quad \text{Io's Stop Influenced Rotation Distance to the Sun (km)}$$

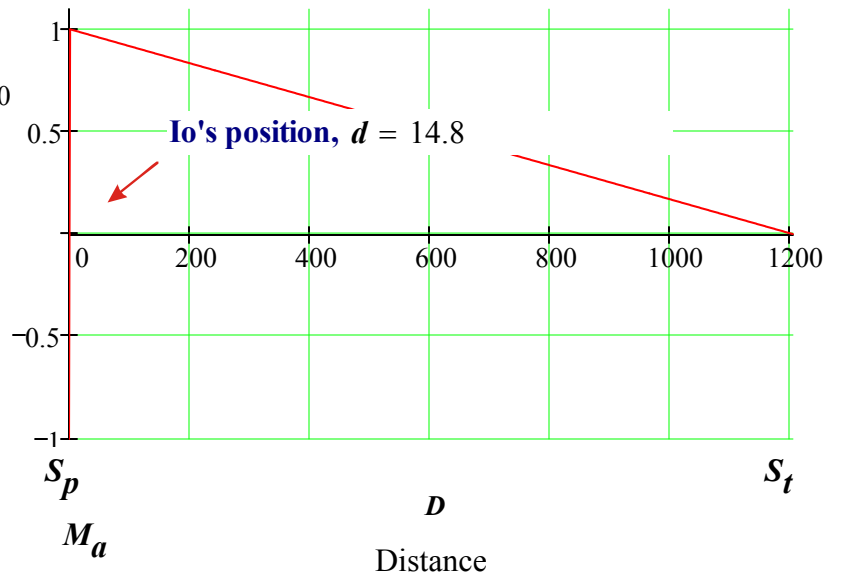
$$d := \frac{a - S_p}{M_a - S_p}$$

$$d = 14.79746369$$

Io's corresponding distance to the Sun relative to S_p on the X axis of the graph

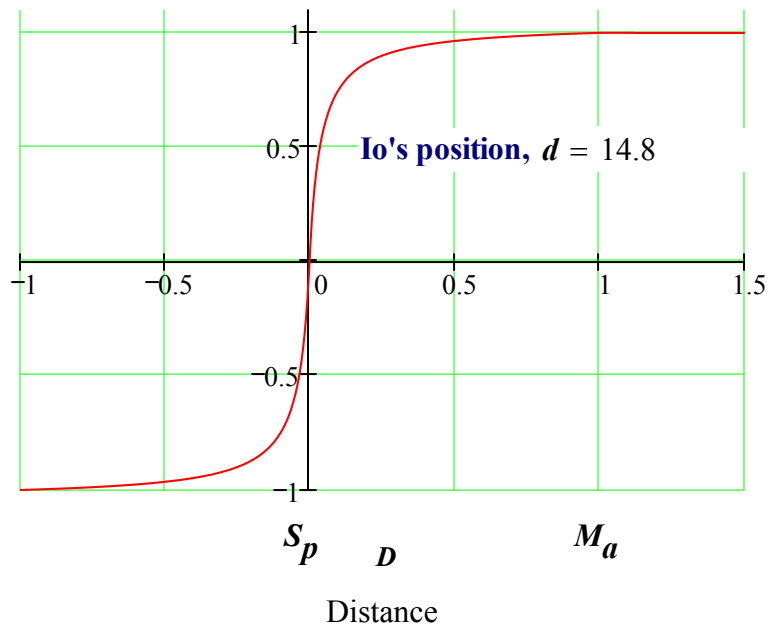
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 & \text{if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

$\omega(d) = 0.98853169$ Io's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 3.13$ Io's Axis Tilt with respect to the Sun (deg)

$$t_r := \text{if} \left(a < M_a, \text{if} \left(\omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90} \right), \text{if} \left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 0.0130699$ Io's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega_2 := \text{if} \left[a < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega_2 = 0.3627418$ **Io's end result Influenced Rotation by the Sun (p.d.)**
(Negative number means the reduction amount from Io's Free Rotation)

Part 3

Io's Total Rotation

$$\omega_s := \sum_{i=1}^2 \omega_i$$

$$\omega_s = 0.3627418 \quad \text{Io's Total Rotation (p.d.)}$$

$$T := \text{if} \left(\omega_1 \leq 0, 0, \text{if} \left(t \leq 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s} \right) \right)$$

$$T = 0.0000$$

Io's Sidereal Rotation Period (day)

If (T = 0 , Io's Synchronous Tropical Rotation)

Observation

$$T_o := 0.0000$$

Io's Sidereal Rotation Period (day)

If (T = 0 , Io's Synchronous Tropical Rotation)

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$$\%Diff = 0.0000$$

Percentage difference between the calculation and the observation