

# Phoebe's Rotation Equations

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## Constants

$$f_R := 2791.826$$

Free Rotation Constant

$$i_M := 0.127036$$

Maximum Influenced Rotation Constant (for planets and moons only)

$$i_{St} := 1.0121647 \cdot 10^{-12}$$

Start Influenced Rotation Distance Constant

$$i_{Ma} := 5.6964797 \cdot 10^{-10}$$

Maximum Influenced Rotation Distance Constant

$$i_{Sp} := 1.0686849 \cdot 10^{-9}$$

Stop Rotation Distance Constant

## Facts

Mass (kg)

$$m_m := 8.2 \cdot 10^{18}$$

## Saturn

$$m_s := 5.6846 \cdot 10^{26}$$

## Sun

$$M := 1.9891 \cdot 10^{30}$$

Density (g/cm<sup>3</sup>)

$$\rho_m := 1.633$$

$$\rho_s := 1.408$$

Axis Tilt (deg)

$$t_m := 26.891$$

$$t := 26.73$$

$$t_s := 7.25$$

Semi-major Axis (km)

$$a_m := 12944700$$

$$a := 1433530000$$

Orbit Eccentricity (deg)

$$e_m := 0.1644$$

$$e_s := 0.0541506$$

Orbit Inclination (deg),  
with respect to equator

$$i_m := 174.986$$

$$i := 5.51$$

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$$\omega_F = 2.51234415$$

Phoebe's Free Rotational Speed (per day)

## Part 1

### Phoebe's Influenced Rotation by the influence of the [Saturn](#)



$$q := a_m \cdot (1 - e_m)$$

$$q = 10816591.3 \quad \text{Phoebe's Perihelion Distance (km)}$$

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 15072808.7 \quad \text{Phoebe's Aphelion Distance (km)}$$

$$i_r := \left( \left| \cos\left(\frac{i_m \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99808669 \quad \text{Phoebe's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m \div i_M}}{\sqrt[3]{M \div m}}$$

$$\omega_{Mi} = 0.02356696 \quad \text{Phoebe's Maximum Influenced Rotational Speed by the Saturn (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$$S_t = 3209792403.5 \quad \text{Phoebe's Start Influenced Rotation Distance to the Saturn (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$$M_a = 5703239.1 \quad \text{Phoebe's Maximum Influenced Rotation Distance to the Saturn (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$$S_p = 3040034.1 \quad \text{Phoebe's Stop Influenced Rotation Distance to the Saturn (km)}$$

**Calculating Phoebe's average distance to the Saturn, if ( $q < S_p < Q$ )**

$$x := \text{if} \left( q < S_p, \text{if} \left( S_p < Q, \frac{S_p - a_m}{e_m}, 0 \right), 0 \right)$$

$x = 0$  X value at Phoebe's orbit intersection with  $S_p$  Boundary (km)

$$b := a_m \sqrt{1 - e_m^2}$$

$b = 12768571.2$  Phoebe's Semi-minor Axis (km)

$$y := b \sqrt{a_m^2 - x^2} \div a_m$$

$y = 12768571.24$  Y value at the Phoebe's orbit intersection with  $S_p$  Boundary (km)

$$\theta := \text{atan} \left( \frac{-x}{y} \right) + \frac{\pi}{2}$$

$\theta = 1.57079633$  Half-angle of the Phoebe's orbit out of  $S_p$  Boundary (rad)

$$P_o := 2 \cdot a_m \cdot \int_0^\pi \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$P_o = 80781571.56$  Phoebe's Orbital Perimeter (km)

$$s_m := a_m \cdot \int_0^\theta \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$s = 20195392.9$  Half of the Phoebe's orbit out of  $S_p$  Boundary (km)

$$a_a := \text{if} \left[ q < S_p, \text{if} \left[ S_p < Q, a_m \frac{\int_{\pi - (s \div a_m)}^\pi \frac{\sqrt{1 - e_m^2 \cdot \cos(E)^2}}{1 \div (1 - e_m \cdot \cos(E))} dE}{\int_{\pi - (s \div a_m)}^\pi \sqrt{1 - e_m^2 \cdot \cos(E)^2} dE}, 0 \right], 0 \right]$$

$a_a = 0$  Phoebe's average distance outside  $S_p$  Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$$n = 0$$

Temporal ratio of the Phoebe's orbit out of  $S_p$  Boundary to the whole orbit

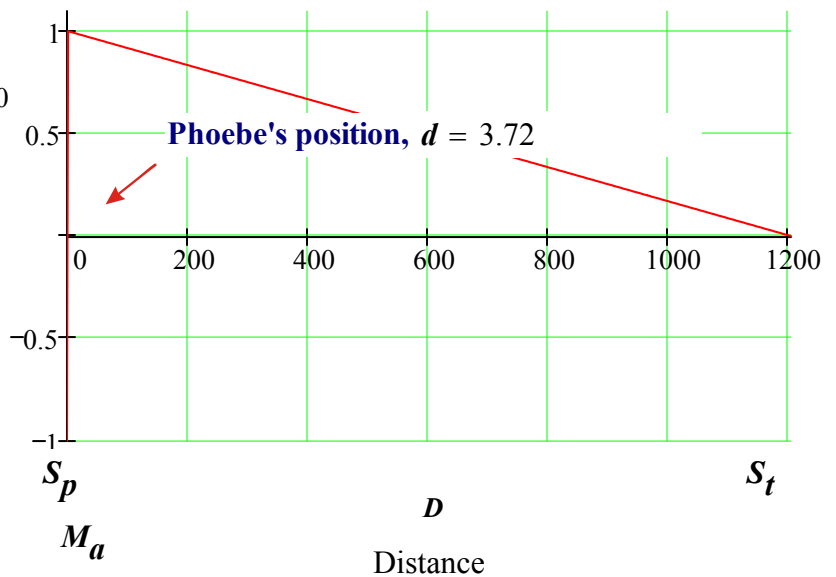
$$d := \text{if} \left( q < S_p, \text{if} \left( S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p} \right), \frac{a_m - S_p}{M_a - S_p} \right)$$

$$d = 3.71907759$$

Phoebe's corresponding distance on x axis of the graph

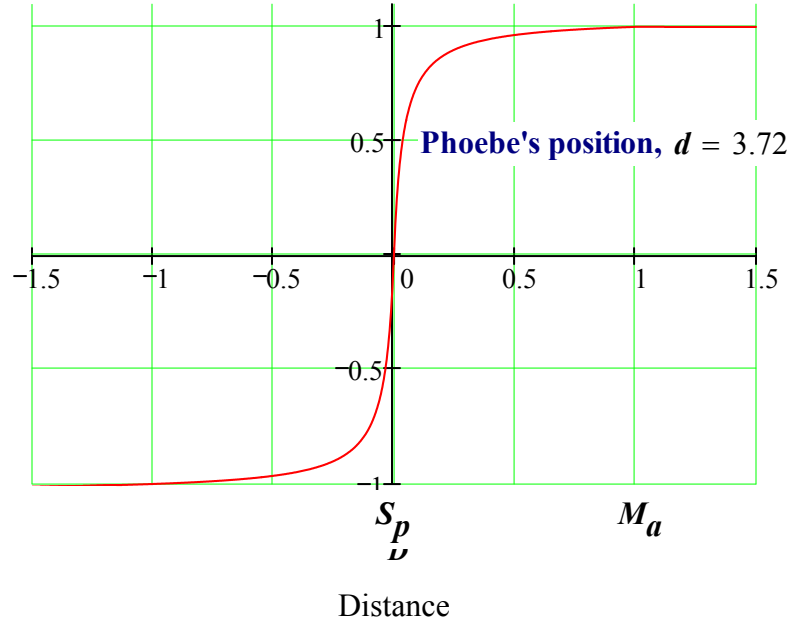
$$\text{Rotation} \left| \begin{array}{l} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 \text{ if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$

Non-proportional Rotation Graph



Left end of the Rotation Graph

$$\text{Rotation} \left| \begin{array}{l} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 \text{ if } 1 < D \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$



$$\omega(d) := \left| \begin{array}{l} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) \text{ if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) \text{ if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 \text{ if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} \end{array} \right.$$

$\omega(d) = 0.99773993$  Phoebe's corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := \text{if} \left[ a_m < M_a, \left( \text{if} \left( \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90} \right) \right), \left( \text{if} \left( \omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right) \right]$$

$t_r = 0.00702563$  Phoebe's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega c_1 := \text{if} \left[ a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[ \omega(d) \cdot (\omega_{Mi} + \omega_F - t_r) \cdot \text{if} (q < S_p, \text{if} (Q > S_p, n, 0), 1) \right] \right]$$

$\omega c_1 = 2.52883222$  Phoebe's end result Influenced Rotation by the Saturn (p.d.)

## Part 2

### Phoebe's Influenced Rotation by the influence of the Sun



$$q_v := a \cdot (1 - e)$$

$$q = 1355903490.4 \quad \text{Phoebe/Saturn's Perihelion Distance to the Sun (km)}$$

$$Q_v := a \cdot (1 + e)$$

$$Q = 1511156509.6 \quad \text{Phoebe/Saturn's Aphelion Distance to the Sun (km)}$$

$$i_r := \left( \left| \cos\left(\frac{i \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99768973 \quad \text{Phoebe/Saturn's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$$\omega_{Mi} = 0.09181936 \quad \text{Phoebe's Maximum Influenced Rotational Speed by the Sun (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$$S_t = 12505688373.1 \quad \text{Phoebe's Start Influenced Rotation Distance to the Sun (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$$M_a = 22220418.6 \quad \text{Phoebe's Maximum Influenced Rotation Distance to the Sun (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

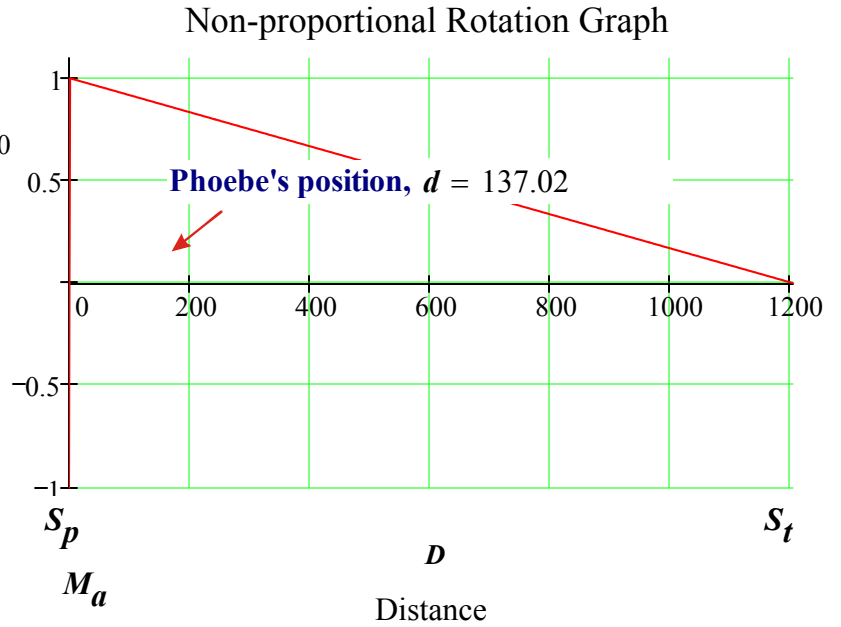
$$S_p = 11844292.3 \quad \text{Phoebe's Stop Influenced Rotation Distance to the Sun (km)}$$

$$d := \frac{a - S_p}{M_a - S_p}$$

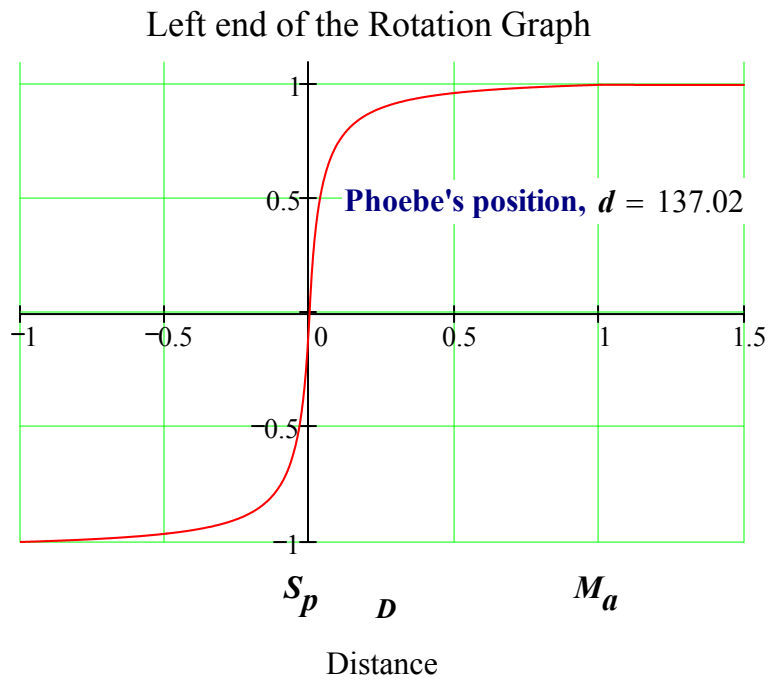
$$d = 137.015074$$

Phoebe's corresponding distance to the Sun relative to  $S_p$  on the X axis of the graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 & \text{if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

$\omega(d) = 0.88694571$  Phoebe's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 53.621$  Phoebe's Axis Tilt with respect to the Sun (deg)

$$t_r := \text{if} \left( a < M_a, \text{if} \left( \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90} \right), \text{if} \left( \omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 0.04852033$  Phoebe's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega_2 := \text{if} \left[ a < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left( q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega_2 = 0.03291846$  **Phoebe's end result Influenced Rotation by the Sun (p.d.)**  
(Negative number means the reduction amount from Phoebe's Free Rotation)



## Part 3

### Phoebe's Total Rotation

$$\omega_s := \sum_{i=1}^2 \omega_i$$

$$\omega_s = 2.56175069 \quad \text{Phoebe's Total Rotation (p.d.)}$$

$$T := \text{if} \left( \omega_1 \leq 0, 0, \text{if} \left( t \leq 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s} \right) \right)$$

$$T = 0.3904$$

**Phoebe's Sidereal Rotation Period (day)**  
**If (T = 0 , Phoebe's Synchronous Tropical Rotation)**

#### Observation

$$T_o := 0.4000$$

**Phoebe's Sidereal Rotation Period (day)**  
**If (T = 0 , Phoebe's Synchronous Tropical Rotation)**

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$$\%Diff = -2.4399$$

Percentage difference between the calculation and the observation