# Prometheus' Rotation Equations

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## Constants

| | |
|---|---|
| $f_R := 2791.826$ | Free Rotation Constant |
| $i_M := 0.127036$ | Maximum Influenced Rotation Constant (for planets and moons only) |
| $i_{St} := 1.0121647 \cdot 10^{-12}$ | Start Influenced Rotation Distance Constant |
| $i_{Ma} := 5.6964797 \cdot 10^{-10}$ | Maximum Influenced Rotation Distance Constant |
| $i_{Sp} := 1.0686849 \cdot 10^{-9}$ | Stop Rotation Distance Constant |

| Facts | Promethe | Saturn | Sun |
|---|---|---|---|
| Mass (kg) | $m_m := 3.3 \cdot 10^{17}$ | $m := 5.6846 \cdot 10^{26}$ | $M := 1.9891 \cdot 10^{30}$ |
| Density (g/cm$^3$) | $\rho_m := 0.63$ | | $\rho_S := 1.408$ |
| Axis Tilt (deg) | $t_m := 0.00$ | $t := 26.73$ | $t_S := 7.25$ |
| Semi-major Axis (km) | $a_m := 139353$ | $a := 1433530000$ | |
| Orbit Eccentricity (deg) | $e_m := 0.0023$ | $e := 0.0541506$ | |
| Orbit Inclination (deg), with respect to equator | $i_m := 0.00$ | $i := 5.51$ | |

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$\omega_F = 2.66566948$ Prometheus' Free Rotational Speed (per day)

# Part 1

## Prometheus' Influenced Rotation by the influence of the Saturn

$$q := a_m \cdot \left(1 - e_m\right)$$

$q = 139032.5$ Prometheus' Perihelion Distance (km)

$$Q := a_m \cdot \left(1 + e_m\right)$$

$Q = 139673.5$ Prometheus' Aphelion Distance (km)

$$i_r := \left(\left|\cos\left(\frac{i_m \cdot \pi}{180}\right)\right| + 1\right) \div 2$$

$i_r = 1$ Prometheus' Orbit Inclination Reduction Factor

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m} \div i_M}{\sqrt[3]{M \div m}}$$

$\omega_{Mi} = 0.01617459$ Prometheus' Maximum Influenced Rotational Speed by the Saturn (p.d.)

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$S_t = 1879600632.1$ Prometheus' Start Influenced Rotation Distance to the Saturn (km)

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$M_a = 3339721.2$ Prometheus' Maximum Influenced Rotation Distance to the Saturn (km)

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$S_p = 1780193$ Prometheus' Stop Influenced Rotation Distance to the Saturn (km)

## Calculating Prometheus' average distance to the Saturn, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a_m}{e_m}, 0\right), 0\right)$$

$x = 0$ X value at Prometheus' orbit intersection with $S_p$ Boundary (km)

$$b := a_m\sqrt{1 - e_m^2}$$

$b = 139352.6$ Prometheus' Semi-minor Axis (km)

$$y := b\sqrt{a_m^2 - x^2} \div a_m$$

$y = 139352.63$ Y value at the Prometheus' orbit intersection with $S_p$ Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Prometheus' orbit out of $S_p$ Boundary (rad)

$$P_o := 2\cdot a_m \cdot \int_0^{\pi} \sqrt{1 - e_m^2 \cdot sin(\theta)^2}\, d\theta$$

$P_o = 875579.56$ Prometheus' Orbital Perimeter (km)

$$s := a_m \cdot \int_0^{\theta} \sqrt{1 - e_m^2 \cdot sin(\theta)^2}\, d\theta$$

$s = 218894.9$ Half of the Prometheus' orbit out of $S_p$ Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a_m \frac{\displaystyle\int_{\pi-(s\div a_m)}^{\pi} \frac{\sqrt{1 - e_m^2\cdot cos(E)^2}}{1 \div \left(1 - e_m\cdot cos(E)\right)}\, dE}{\displaystyle\int_{\pi-(s\div a_m)}^{\pi} \sqrt{1 - e_m^2\cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Prometheus' average distance outside $S_p$ Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^{\,3}}{a^3}}$$

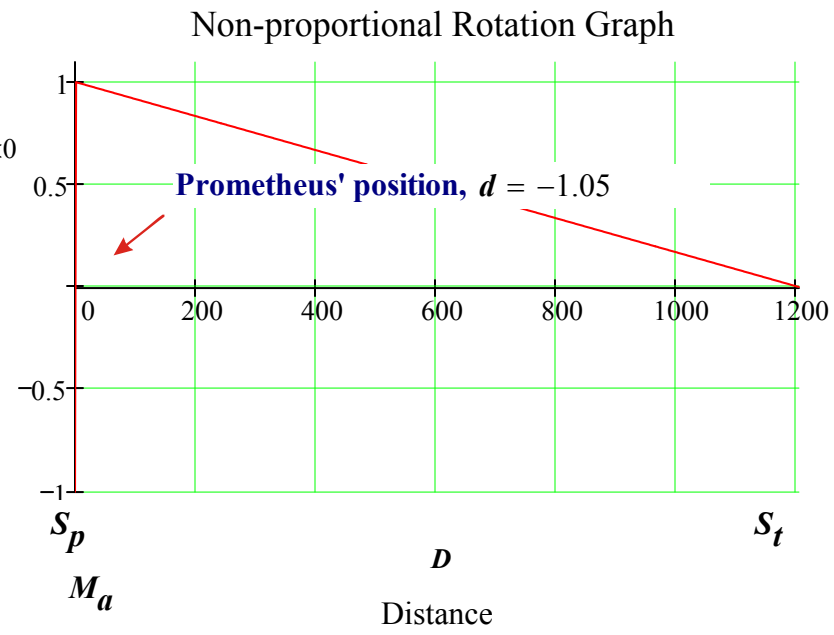
$n = 0$

Temporal ratio of the Prometheus' orbit out of $S_p$ Boundary to the whole orbit

$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p}\right), \frac{a_m - S_p}{M_a - S_p}\right)$$

$d = -1.05213875$ Prometheus' corresponding distance on x axis of the graph

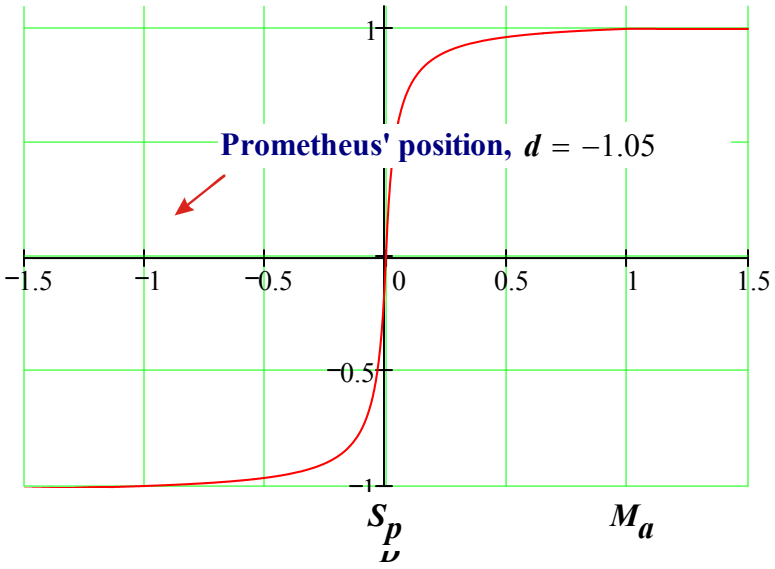
$$\text{Rotation} := \begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



Rotation

$$\begin{cases} -1-\dfrac{0.04}{D-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } D<0 \\ \dfrac{-0.04}{D+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le D\le 1 \\ \dfrac{-D+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<D \end{cases}$$

Left end of the Rotation Graph




Distance

$$\omega(d) := \begin{cases} -1-\dfrac{0.04}{d-\left(\sqrt{0.29}-0.5\right)}-\left(\sqrt{0.29}-0.5\right) & \text{if } d<0 \\ \dfrac{-0.04}{d+\left(\sqrt{0.29}-0.5\right)}+\left(0.5+\sqrt{0.29}\right) & \text{if } 0\le d\le 1 \\ \dfrac{-d+1}{\dfrac{S_t-M_a}{M_a-S_p}}+1 & \text{if } 1<d \end{cases}$$

$\omega(d) = -1.00184128$ Prometheus' corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := \text{if}\left[a_m < M_a, \left(\text{if}\left(\omega_{Mi} > \omega_F, \frac{t_m\cdot\omega_F}{90}, \frac{t_m\cdot\omega_{Mi}}{90}\right)\right), \left(\text{if}\left(\omega(d)\cdot\omega_{Mi} > \omega_F, \frac{t_m\cdot\omega_F}{90}, \frac{t_m\cdot\omega(d)\cdot\omega_{Mi}}{90}\right)\right)\right]$$

$t_r = 0$ Prometheus' Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega c_1 := \text{if}\left[a_m > M_a, \omega(d)\cdot\omega_{Mi}+\omega_F-t_r, \left[\omega(d)\cdot\left(\omega_{Mi}+\omega_F-t_r\right)\cdot\text{if}\left(q < S_p, \text{if}\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega c_1 = 0$ **Prometheus' end result Influenced Rotation by the Saturn (p.d.)**

# Part 2

## Prometheus' Influenced Rotation by the influence of the Sun

$q := a \cdot (1 - e)$

$q = 1355903490.4$ Prometheus/Saturn's Perihelion Distance to the Sun (km)

$Q := a \cdot (1 + e)$

$Q = 1511156509.6$ Prometheus/Saturn's Aphelion Distance to the Sun (km)

$$i_r := \left( \left| \cos\left( \frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99768973$ Prometheus/Saturn's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$\omega_{Mi} = 0.06299778$ Prometheus' Maximum Influenced Rotational Speed by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$S_t = 7320784401$ Prometheus' Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 13007752$ Prometheus' Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

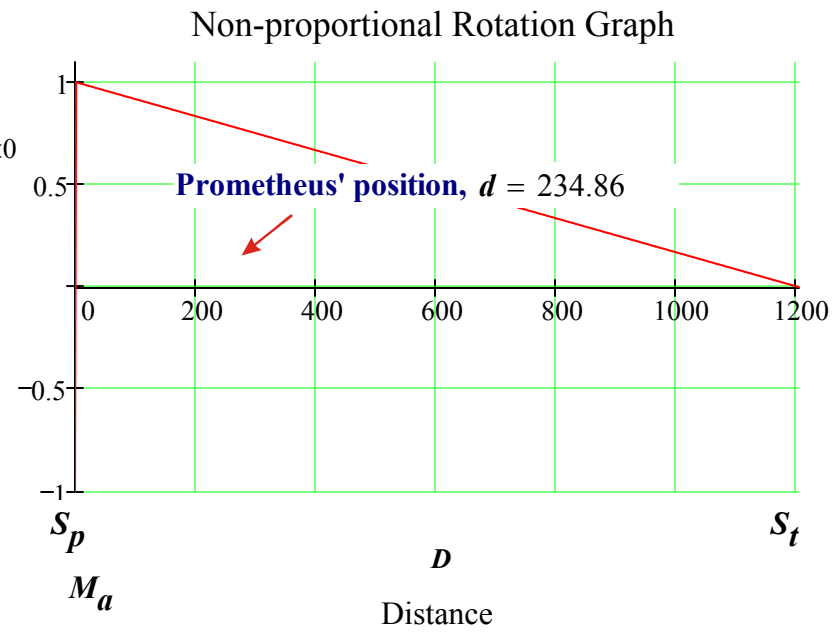
$S_p = 6933605.5$ Prometheus' Stop Influenced Rotation Distance to the Sun (km)

$$d := \frac{a - S_p}{M_a - S_p}$$

$d = 234.86367904$ Prometheus' corresponding distance to the Sun relative to $S_p$ on the X axis of the graph

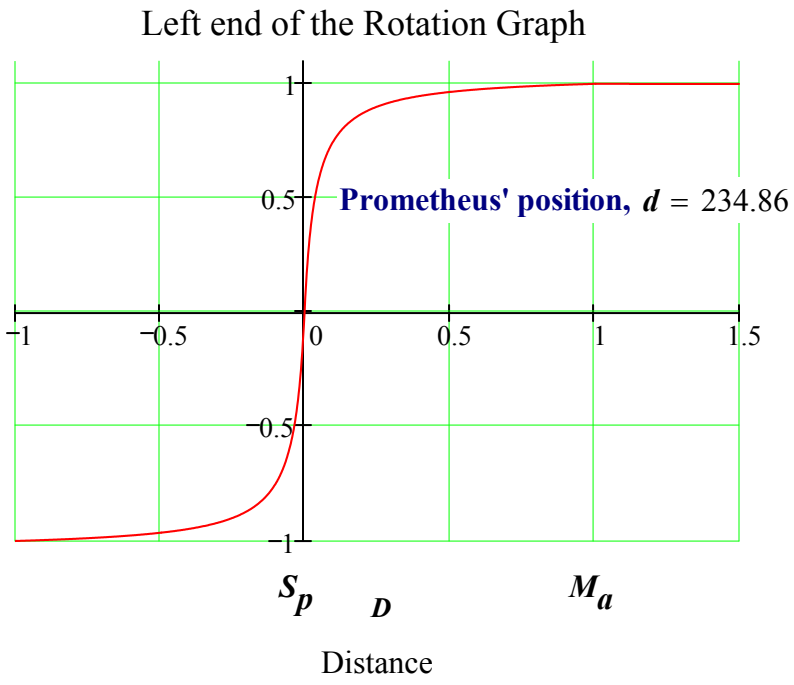
Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$




Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\ \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\ \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\frac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.80561499$ Prometheus' corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$t_{m2} := t_m + t$

$t_{m2} = 26.73$ Prometheus' Axis Tilt with respect to the Sun (deg)

$t_r := if\left(a < M_a, if\left(\omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90}\right), if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$

$t_r = 0.01507333$ Prometheus' Maximum and Free Rotational Speed Reduction by Axis Tilt

$\omega c_2 := if\left[a < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), if\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$

$\omega c_2 = 0.03567862$ **Prometheus' end result Influenced Rotation by the Sun (p.d.)**

(Negative number means the reduction amount from Prometheus' Free Rotation)

# Part 3

## Prometheus' Total Rotation

$$\omega_S := \sum_{i=1}^{2} \omega c_i$$

$\omega_S = 0.03567862$ Prometheus' Total Rotation (p.d.)

$$T := if\left(\omega c_1 \le 0\,,0\,,if\left(t \le 90\,,\frac{1}{\omega_S}\,,\frac{-1}{\omega_S}\right)\right)$$

$T = 0.0000$

**Prometheus' Sidereal Rotation Period (day)**
**If (T = 0 , Prometheus' Synchronous Tropical Rotation)**

### Observation

$T_o := 0.0000$

**Prometheus' Sidereal Rotation Period (day)**
**If (T = 0 , Prometheus' Synchronous Tropical Rotation)**

$$\%Diff := \frac{\left(T - T_o\right)\cdot 200}{T + T_o}$$

$\%Diff = 0.0000$ Percentage deference between the calculation and the observation