# Rosalind's Rotation Equations

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## Constants

| | |
|---|---|
| $f_R := 2791.826$ | Free Rotation Constant |
| $i_M := 0.127036$ | Maximum Influenced Rotation Constant (for planets and moons only) |
| $i_{St} := 1.0121647 \cdot 10^{-12}$ | Start Influenced Rotation Distance Constant |
| $i_{Ma} := 5.6964797 \cdot 10^{-10}$ | Maximum Influenced Rotation Distance Constant |
| $i_{Sp} := 1.0686849 \cdot 10^{-9}$ | Stop Rotation Distance Constant |

| Facts | Rosalind | Saturn | Sun |
|---|---|---|---|
| Mass (kg) | $m_m := 2.54 \cdot 10^{17}$ | $m := 8.6832 \cdot 10^{25}$ | $M := 1.9891 \cdot 10^{30}$ |
| Density (g/cm³) | $\rho_m := 1.300$ | | $\rho_s := 1.408$ |
| Axis Tilt (deg) | $t_m := 0.000$ | $t := 97.77$ | $t_s := 7.25$ |
| Semi-major Axis (km) | $a_m := 69940$ | $a := 2872460000$ | |
| Orbit Eccentricity (deg) | $e_m := 0.0001$ | $e := 0.04716771$ | |
| Orbit Inclination (deg), with respect to equator | $i_m := 0.2790$ | $i := 6.48$ | |

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$\omega_F = 3.99994854$ Rosalind's Free Rotational Speed (per day)

# Part 1

## Rosalind's Influenced Rotation by the influence of the Saturn

$$q := a_m \cdot (1 - e_m)$$

$q = 69933$ Rosalind's Perihelion Distance (km)

$$Q := a_m \cdot (1 + e_m)$$

$Q = 69947$ Rosalind's Aphelion Distance (km)

$$i_r := \left( \left| \cos\left( \frac{i_m \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99999407$ Rosalind's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m} \div i_M}{\sqrt[3]{M \div m}}$$

$\omega_{Mi} = 0.01003342$ Rosalind's Maximum Influenced Rotational Speed by the Saturn (p.d.)

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$S_t = 1315573444.3$ Rosalind's Start Influenced Rotation Distance to the Saturn (km)

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$M_a = 2337543.7$ Rosalind's Maximum Influenced Rotation Distance to the Saturn (km)

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$S_p = 1245995.9$ Rosalind's Stop Influenced Rotation Distance to the Saturn (km)

## Calculating Rosalind's average distance to the Saturn, if $(q < S_p < Q)$

$$x := if\left(q < S_p, if\left(S_p < Q, \frac{S_p - a_m}{e_m}, 0\right), 0\right)$$

$x = 0$ X value at Rosalind's orbit intersection with $S_p$ Boundary (km)

$$b := a_m \sqrt{1 - e_m^{\,2}}$$

$b = 69940$ Rosalind's Semi-minor Axis (km)

$$y := b\sqrt{a_m^{\,2} - x^2} \div a_m$$

$y = 69940$ Y value at the Rosalind's orbit intersection with $S_p$ Boundary (km)

$$\theta := atan\left(\frac{-x}{y}\right) + \frac{\pi}{2}$$

$\theta = 1.57079633$ Half-angle of the Rosalind's orbit out of $S_p$ Boundary (rad)

$$P_o := 2 \cdot a_m \cdot \int_0^{\pi} \sqrt{1 - e_m^{\,2} \cdot sin(\theta)^2}\, d\theta$$

$P_o = 439445.98$ Rosalind's Orbital Perimeter (km)

$$s := a_m \cdot \int_0^{\theta} \sqrt{1 - e_m^{\,2} \cdot sin(\theta)^2}\, d\theta$$

$s = 109861.5$ Half of the Rosalind's orbit out of $S_p$ Boundary (km)

$$a_a := if\left[q < S_p, if\left[S_p < Q, a_m \frac{\displaystyle\int_{\pi-(s \div a_m)}^{\pi} \frac{\sqrt{1 - e_m^{\,2} \cdot cos(E)^2}}{1 \div \left(1 - e_m \cdot cos(E)\right)}\, dE}{\displaystyle\int_{\pi-(s \div a_m)}^{\pi} \sqrt{1 - e_m^{\,2} \cdot cos(E)^2}\, dE}, 0\right], 0\right]$$

$a_a = 0$ Rosalind's average distance outside $S_p$ Boundary (km)

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^{\,3}}{a^3}}$$

$n = 0$ Temporal ratio of the Rosalind's orbit out of $S_p$ Boundary to the whole orbit

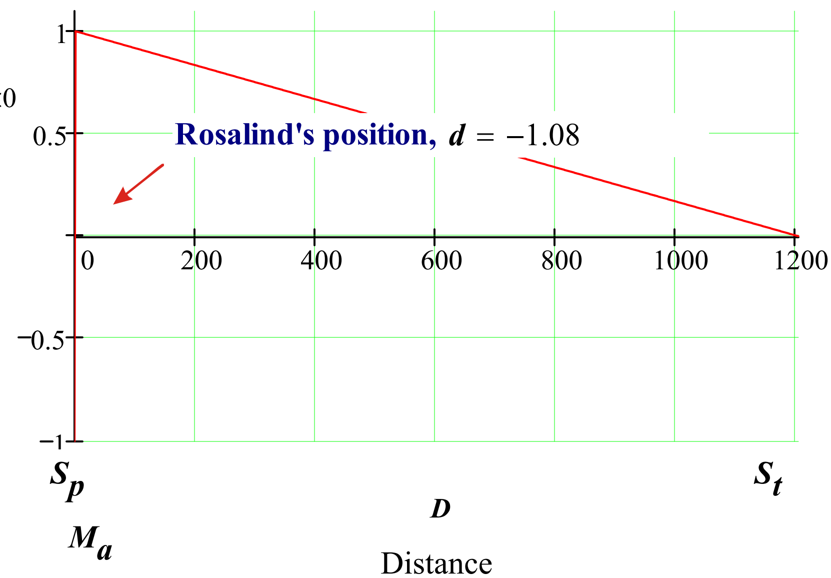
$$d := if\left(q < S_p, if\left(S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p}\right), \frac{a_m - S_p}{M_a - S_p}\right)$$

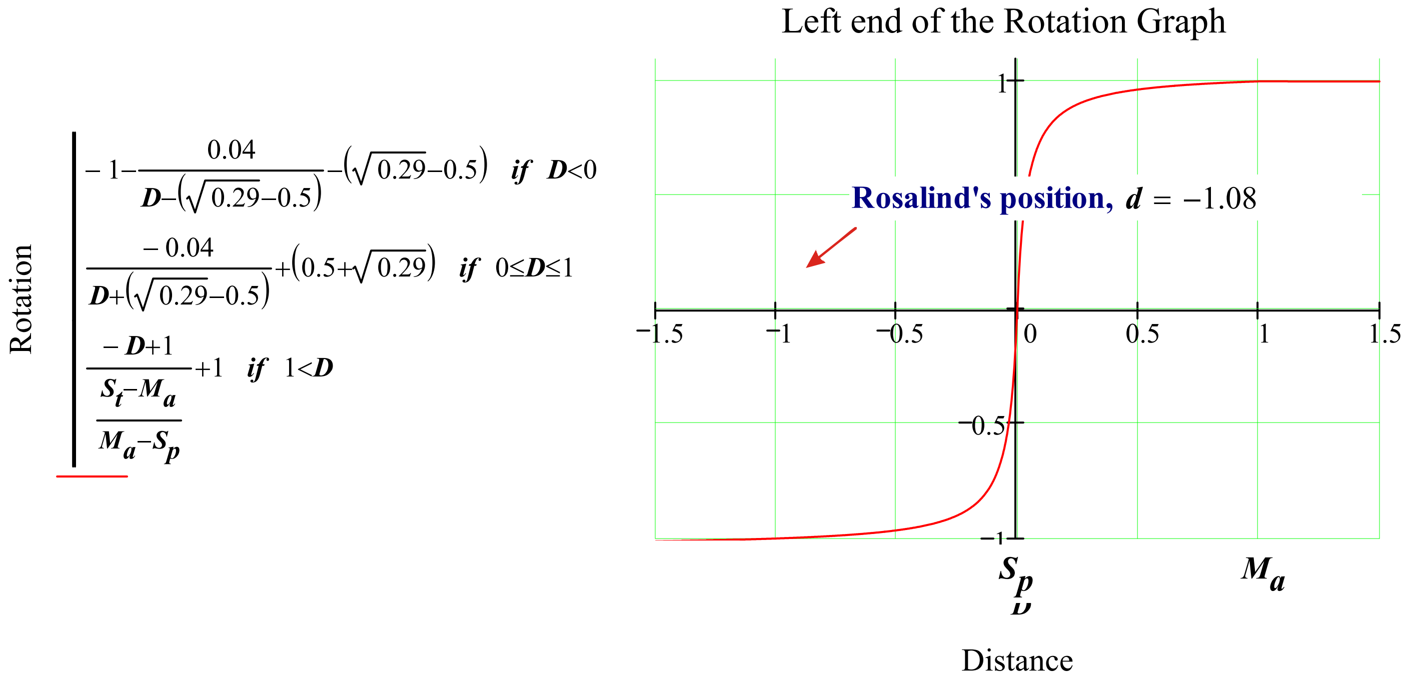
$d = -1.07742046$ Rosalind's corresponding distance on x axis of the graph

Non-proportional Rotation Graph

Rotation

$$\begin{cases} -1 - \dfrac{0.04}{D - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } D < 0 \\[2ex] \dfrac{-0.04}{D + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le D \le 1 \\[2ex] \dfrac{-D + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < D \end{cases}$$






$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = -1.00267216$ Rosalind's corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := if\left[a_m < M_a, \left(if\left(\omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90}\right)\right), \left(if\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)\right]$$

$t_r = 0$ Rosalind's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega c_1 := if\left[a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[\omega(d) \cdot \left(\omega_{Mi} + \omega_F - t_r\right) \cdot if\left(q < S_p, if\left(Q > S_p, n, 0\right), 1\right)\right]\right]$$

$\omega c_1 = 0$ **Rosalind's end result Influenced Rotation by the Saturn (p.d.)**

## Part 2

### Rosalind's Influenced Rotation by the influence of the Sun

$q := a \cdot (1 - e)$

$q = 2736972639.7$ Rosalind/Saturn's Perihelion Distance to the Sun (km)

$Q := a \cdot (1 + e)$

$Q = 3007947360.3$ Rosalind/Saturn's Aphelion Distance to the Sun (km)

$$i_r := \left( \left| cos\left( \frac{i \cdot \pi}{180} \right) \right| + 1 \right) \div 2$$

$i_r = 0.99680566$ Rosalind/Saturn's Orbit Inclination Reduction Factor

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$\omega_{Mi} = 0.05344174$ Rosalind's Maximum Influenced Rotational Speed by the Sun (p.d.)

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$S_t = 7007236010.3$ Rosalind's Start Influenced Rotation Distance to the Sun (km)

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$M_a = 12450631.5$ Rosalind's Maximum Influenced Rotation Distance to the Sun (km)

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

$S_p = 6636640$ Rosalind's Stop Influenced Rotation Distance to the Sun (km)

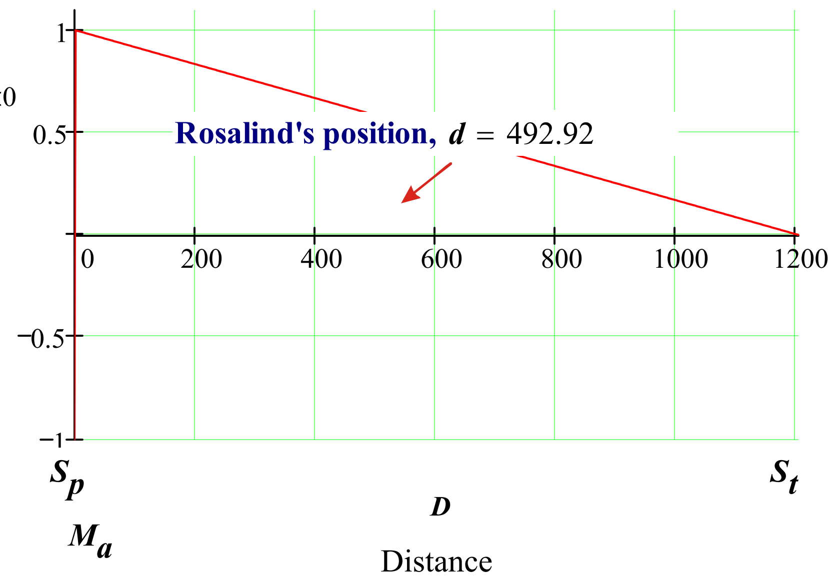
$$d := \frac{a - S_p}{M_a - S_p}$$

$d = 492.91839731$ Rosalind's corresponding distance to the Sun relative to $S_p$ on the X axis of the graph

Rotation

$$\begin{cases} -1 - \frac{0.04}{D-(\sqrt{0.29}-0.5)} - (\sqrt{0.29}-0.5) & \text{if } D<0 \\ \frac{-0.04}{D+(\sqrt{0.29}-0.5)} + (0.5+\sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \frac{-D+1}{\frac{S_t-M_a}{M_a-S_p}} + 1 & \text{if } 1<D \end{cases}$$

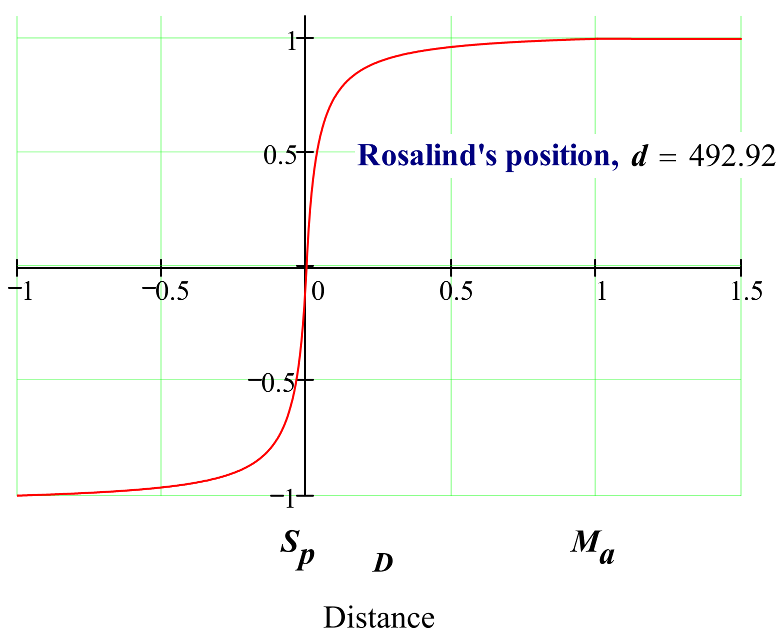
Non-proportional Rotation Graph




Rotation

$$\begin{cases} -1 - \frac{0.04}{D-(\sqrt{0.29}-0.5)} - (\sqrt{0.29}-0.5) & \text{if } D<0 \\ \frac{-0.04}{D+(\sqrt{0.29}-0.5)} + (0.5+\sqrt{0.29}) & \text{if } 0 \le D \le 1 \\ \frac{-D+1}{\frac{S_t-M_a}{M_a-S_p}} + 1 & \text{if } 1<D \end{cases}$$

Left end of the Rotation Graph



$$\omega(d) := \begin{cases} -1 - \dfrac{0.04}{d - \left(\sqrt{0.29} - 0.5\right)} - \left(\sqrt{0.29} - 0.5\right) & \text{if } d < 0 \\ \dfrac{-0.04}{d + \left(\sqrt{0.29} - 0.5\right)} + \left(0.5 + \sqrt{0.29}\right) & \text{if } 0 \le d \le 1 \\ \dfrac{-d + 1}{\dfrac{S_t - M_a}{M_a - S_p}} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = 0.59112264$ Rosalind's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 97.77$ Rosalind's Axis Tilt with respect to the Sun (deg)

$$t_r := \mathrm{if}\left(a < M_a, \mathrm{if}\left(\omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90}\right), \mathrm{if}\left(\omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90}\right)\right)$$

$t_r = 0.03431794$ Rosalind's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega c_2 := \mathrm{if}\left[a < M_a, \omega(d) \cdot \left(\omega_{Mi} - t_r\right), \mathrm{if}\left(q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0\right)\right]$$

$\omega c_2 = -0.00272732$ **Rosalind's end result Influenced Rotation by the Sun (p.d.)**

(Negative number means the reduction amount from Rosalind's Free Rotation)

# Part 3

## Rosalind's Total Rotation

$$\omega_s := \sum_{i=1}^{2} \omega c_i$$

$\omega_s = -0.00272732$ Rosalind's Total Rotation (p.d.)

$$T := if\left(\omega c_1 \le 0, 0, if\left(t \le 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s}\right)\right)$$

$T = 0.0000$

**Rosalind's Sidereal Rotation Period (day)**
**If (T = 0 , Rosalind's Synchronous Tropical Rotation)**

**Observation**

$T_o := 0.0000$

**Rosalind's Sidereal Rotation Period (day)**
**If ($T$ = 0 , Rosalind's Synchronous Tropical Rotation)**

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$\%Diff = 0.0000$ Percentage deference between the calculation and the observation