

# Titania's Rotation Equations

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## Constants

$f_R := 2791.826$	Free Rotation Constant
$i_M := 0.127036$	Maximum Influenced Rotation Constant (for planets and moons only)
$i_{St} := 1.0121647 \cdot 10^{-12}$	Start Influenced Rotation Distance Constant
$i_{Ma} := 5.6964797 \cdot 10^{-10}$	Maximum Influenced Rotation Distance Constant
$i_{Sp} := 1.0686849 \cdot 10^{-9}$	Stop Rotation Distance Constant

## Facts

	<u>Titania</u>	<u>Saturn</u>	<u>Sun</u>
Mass (kg)	$m_m := 3.52 \cdot 10^{21}$	$m_s := 8.6832 \cdot 10^{25}$	$M := 1.9891 \cdot 10^{30}$
Density (g/cm <sup>3</sup> )	$\rho_m := 1.715$		$\rho_s := 1.408$
Axis Tilt (deg)	$t_m := 0.020$	$t := 97.77$	$t_s := 7.25$
Semi-major Axis (km)	$a_m := 436300$	$a := 2872460000$	
Orbit Eccentricity (deg)	$e_m := 0.0011$	$e_s := 0.04716771$	
Orbit Inclination (deg), with respect to equator	$i_m := 0.079$	$i := 6.48$	

$$\omega_F := f_R \div \sqrt[6]{m_m} \cdot \sqrt[2]{\rho_m}$$

$$\omega_F = 0.93741083$$

Titania's Free Rotational Speed (per day)

## Part 1

### Titania's Influenced Rotation by the influence of the [Saturn](#)



$$q := a_m \cdot (1 - e_m)$$

$$q = 435820.1 \quad \text{Titania's Perihelion Distance (km)}$$

$$Q := a_m \cdot (1 + e_m)$$

$$Q = 436779.9 \quad \text{Titania's Aphelion Distance (km)}$$

$$i_r := \left( \left| \cos\left(\frac{i_m \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99999952 \quad \text{Titania's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div \sqrt[6]{\rho_m \div i_M}}{\sqrt[3]{M \div m}}$$

$$\omega_{Mi} = 0.04695487 \quad \text{Titania's Maximum Influenced Rotational Speed by the Saturn (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{St}}{\sqrt[3]{M \div m}}$$

$$S_t = 6447633120.5 \quad \text{Titania's Start Influenced Rotation Distance to the Saturn (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Ma}}{\sqrt[3]{M \div m}}$$

$$M_a = 11456315.1 \quad \text{Titania's Maximum Influenced Rotation Distance to the Saturn (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div m} \div i_{Sp}}{\sqrt[3]{M \div m}}$$

$$S_p = 6106633.2 \quad \text{Titania's Stop Influenced Rotation Distance to the Saturn (km)}$$

## Calculating Titania's average distance to the Saturn, if ( $q < S_p < Q$ )

$$x := \text{if} \left( q < S_p, \text{if} \left( S_p < Q, \frac{S_p - a_m}{e_m}, 0 \right), 0 \right)$$

$$x = 0 \quad \text{X value at Titania's orbit intersection with } S_p \text{ Boundary (km)}$$

$$b := a_m \sqrt{1 - e_m^2}$$

$$b = 436299.7 \quad \text{Titania's Semi-minor Axis (km)}$$

$$y := b \sqrt{a_m^2 - x^2} \div a_m$$

$$y = 436299.74 \quad \text{Y value at the Titania's orbit intersection with } S_p \text{ Boundary (km)}$$

$$\theta := \text{atan} \left( \frac{-x}{y} \right) + \frac{\pi}{2}$$

$$\theta = 1.57079633 \quad \text{Half-angle of the Titania's orbit out of } S_p \text{ Boundary (rad)}$$

$$P_o := 2 \cdot a_m \cdot \int_0^\pi \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$$P_o = 2741352.92 \quad \text{Titania's Orbital Perimeter (km)}$$

$$s_m := a_m \cdot \int_0^\theta \sqrt{1 - e_m^2 \cdot \sin(\theta)^2} d\theta$$

$$s = 685338.2 \quad \text{Half of the Titania's orbit out of } S_p \text{ Boundary (km)}$$

$$a_a := \text{if} \left[ q < S_p, \text{if} \left[ S_p < Q, a_m \frac{\int_{\pi - (s \div a_m)}^\pi \frac{\sqrt{1 - e_m^2 \cdot \cos(E)^2}}{1 \div (1 - e_m \cdot \cos(E))} dE}{\int_{\pi - (s \div a_m)}^\pi \sqrt{1 - e_m^2 \cdot \cos(E)^2} dE}, 0 \right], 0 \right]$$

$$a_a = 0 \quad \text{Titania's average distance outside } S_p \text{ Boundary (km)}$$

$$n := \frac{2 \cdot s}{P_o} \cdot \sqrt{\frac{a_a^3}{a^3}}$$

$$n = 0$$

Temporal ratio of the Titania's orbit out of  $S_p$  Boundary to the whole orbit

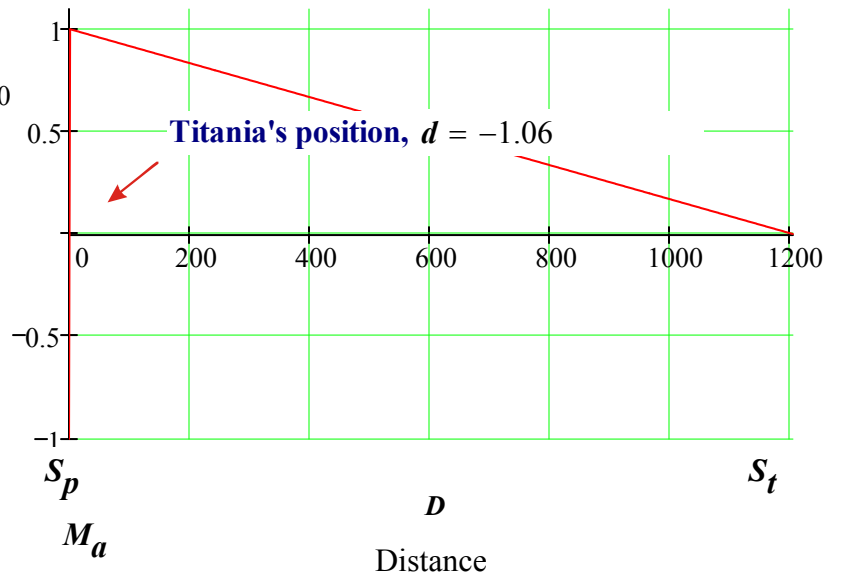
$$d := \text{if} \left( q < S_p, \text{if} \left( S_p < Q, \frac{a_a - S_p}{M_a - S_p}, \frac{a_m - S_p}{M_a - S_p} \right), \frac{a_m - S_p}{M_a - S_p} \right)$$

$$d = -1.05993837$$

Titania's corresponding distance on x axis of the graph

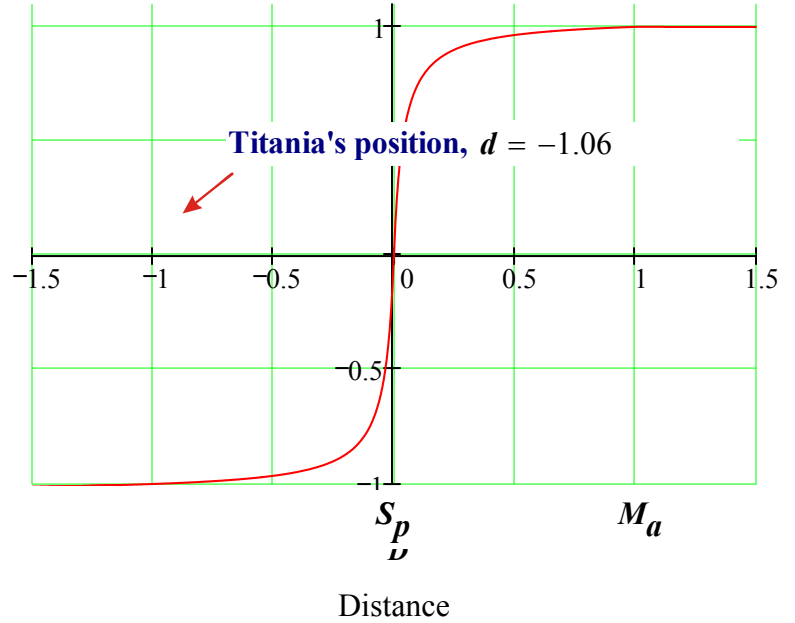
$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$

Non-proportional Rotation Graph



### Left end of the Rotation Graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 & \text{if } 1 < d \end{cases}$$

$\omega(d) = -1.00210169$  Titania's corresponding Influenced Rotation by the Saturn on the Y axis of the graph

$$t_r := \text{if} \left[ a_m < M_a, \left( \text{if} \left( \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega_{Mi}}{90} \right) \right), \left( \text{if} \left( \omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_m \cdot \omega_F}{90}, \frac{t_m \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right) \right]$$

$t_r = 0.00001043$  Titania's Maximum and Free Rotational Speed Reduction by Axis Tilt Degree

$$\omega_{c1} := \text{if} \left[ a_m > M_a, \omega(d) \cdot \omega_{Mi} + \omega_F - t_r, \left[ \omega(d) \cdot (\omega_{Mi} + \omega_F - t_r) \cdot \text{if} (q < S_p, \text{if} (Q > S_p, n, 0), 1) \right] \right]$$

$\omega_{c1} = 0$  Titania's end result Influenced Rotation by the Saturn (p.d.)

## Part 2

### Titania's Influenced Rotation by the influence of the Sun



$$q_v := a \cdot (1 - e)$$

$$q = 2736972639.7 \quad \text{Titania/Saturn's Perihelion Distance to the Sun (km)}$$

$$Q_v := a \cdot (1 + e)$$

$$Q = 3007947360.3 \quad \text{Titania/Saturn's Aphelion Distance to the Sun (km)}$$

$$i_r := \left( \left| \cos\left(\frac{i \cdot \pi}{180}\right) \right| + 1 \right) \div 2$$

$$i_r = 0.99680566 \quad \text{Titania/Saturn's Orbit Inclination Reduction Factor}$$

$$\omega_{Mi} := \frac{\sqrt[6]{m_m \cdot i_r \div M} \div \sqrt[6]{\rho_m}}{i_M}$$

$$\omega_{Mi} = 0.25009898 \quad \text{Titania's Maximum Influenced Rotational Speed by the Sun (p.d.)}$$

$$S_t := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{St}}$$

$$S_t = 34342473330.9 \quad \text{Titania's Start Influenced Rotation Distance to the Sun (km)}$$

$$M_a := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Ma}}$$

$$M_a = 61020561.9 \quad \text{Titania's Maximum Influenced Rotation Distance to the Sun (km)}$$

$$S_p := \frac{\sqrt[6]{m_m \cdot i_r \div M}}{i_{Sp}}$$

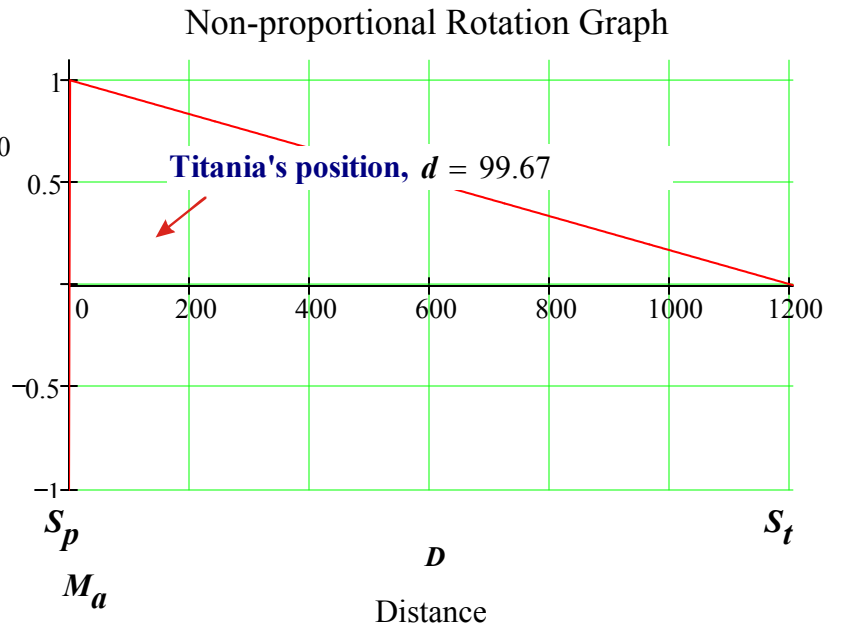
$$S_p = 32526181.7 \quad \text{Titania's Stop Influenced Rotation Distance to the Sun (km)}$$

$$d := \frac{a - S_p}{M_a - S_p}$$

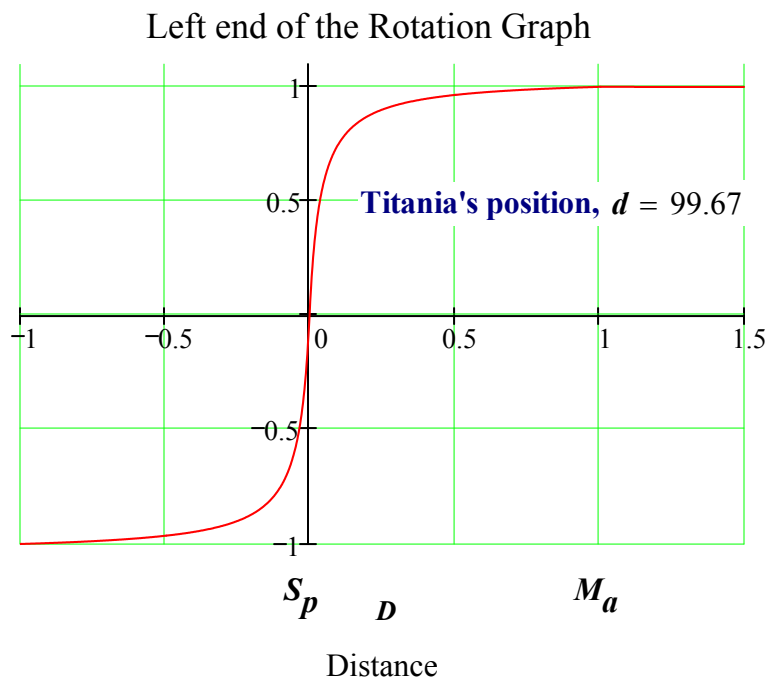
$$d = 99.66645339$$

Titania's corresponding distance to the Sun relative to  $S_p$  on the X axis of the graph

$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\text{Rotation} = \begin{cases} -1 - \frac{0.04}{D - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } D < 0 \\ \frac{-0.04}{D + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq D \leq 1 \\ \frac{-D + 1}{S_t - M_a} + 1 & \text{if } 1 < D \end{cases}$$



$$\omega(d) := \begin{cases} -1 - \frac{0.04}{d - (\sqrt{0.29} - 0.5)} - (\sqrt{0.29} - 0.5) & \text{if } d < 0 \\ \frac{-0.04}{d + (\sqrt{0.29} - 0.5)} + (0.5 + \sqrt{0.29}) & \text{if } 0 \leq d \leq 1 \\ \frac{-d + 1}{S_t - M_a} + 1 & \text{if } 1 < d \\ \frac{M_a - S_p}{M_a - S_p} & \end{cases}$$

$\omega(d) = 0.91798949$  Titania's corresponding Influenced Rotational Speed by the Sun on Y axis of the graph

$$t_{m2} := t_m + t$$

$t_{m2} = 97.79$  Titania's Axis Tilt with respect to the Sun (deg)

$$t_r := \text{if} \left( a < M_a, \text{if} \left( \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega_{Mi}}{90} \right), \text{if} \left( \omega(d) \cdot \omega_{Mi} > \omega_F, \frac{t_{m2} \cdot \omega_F}{90}, \frac{t_{m2} \cdot \omega(d) \cdot \omega_{Mi}}{90} \right) \right)$$

$t_r = 0.24946037$  Titania's Maximum and Free Rotational Speed Reduction by Axis Tilt

$$\omega_2 := \text{if} \left[ a < M_a, \omega(d) \cdot (\omega_{Mi} - t_r), \text{if} \left( q < S_t, \omega(d) \cdot \omega_{Mi} - t_r, 0 \right) \right]$$

$\omega_2 = -0.01987214$  **Titania's end result Influenced Rotation by the Sun (p.d.)**  
(Negative number means the reduction amount from Titania's Free Rotation)



## Part 3

### Titania's Total Rotation

$$\omega_s := \sum_{i=1}^2 \omega_i$$

$$\omega_s = -0.01987214 \quad \text{Titania's Total Rotation (p.d.)}$$

$$T := \text{if} \left( \omega_1 \leq 0, 0, \text{if} \left( t \leq 90, \frac{1}{\omega_s}, \frac{-1}{\omega_s} \right) \right)$$

$$T = 0.0000$$

**Titania's Sidereal Rotation Period (day)**  
**If (T = 0 , Titania's Synchronous Tropical Rotation)**

### Observation

$$T_o := 0.0000$$

**Titania's Sidereal Rotation Period (day)**  
**If (T = 0 , Titania's Synchronous Tropical Rotation)**

$$\%Diff := \frac{(T - T_o) \cdot 200}{T + T_o}$$

$$\%Diff = 0.0000$$

Percentage difference between the calculation and the observation